St. Luke's C. of E. Primary School
Calculation Policy

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Sense of Number Maths Consultants

Contents:

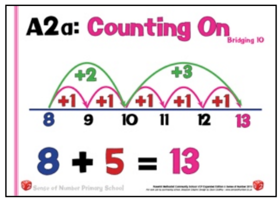
Overview of Calculation Approaches

Early Years into KS1

- Visualisation to secure understanding of the number system, especially the use of place value resources such as Base 10, Numicon, 100 Squares and abaci.
- Secure understanding of numbers to 10, using resources such as Numicon, Tens Frames, fingers and multi-link.
- Subitising to begin making links between the different images of a number and their links to calculation.
- Practical, oral and mental activities to understand calculation.
- Personal methods of recording.

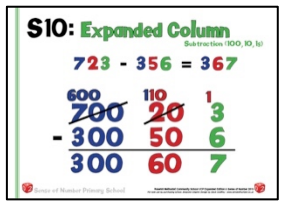
Key Stage 1

- Introduce signs and symbols ***(+ , -, x, ÷ in Year 1 and <, > signs in Year 2)***
- Extended visualisation to secure understanding of the number system beyond 100, especially the use of place value resources such as Base 10, Place Value Charts & Grids, Number Grids, Arrow Cards and Place Value Counters.
- Further work on subitising and Tens Frames to develop basic calculation understanding, supported by Numicon and multi-link.
- Continued use of practical apparatus to support the early teaching of 2-digit calculation. For example, using Base 10 or Numicon to demonstrate partitioning and exchanging before these methods are taught as jottings / number sentences.
- Methods of recording / jottings to support calculation (e.g. partitioning or counting on).
- Use images such as empty number lines to support mental and informal calculation.



Year 3

- Continued use of practical apparatus, especially Place Value Counters, Base 10 and Numicon to visualise written / column methods before and as they are actually taught as procedures.
- Continued use of mental methods and jottings for 2 and 3 digit calculations.
- Introduction to more efficient informal written methods / jottings including expanded methods and efficient use of number lines (especially for subtraction).
- Column methods, where appropriate, for 3 digit additions and subtractions.



Years 4-6

- Continued use of mental methods for any appropriate calculation up to 6 digits.
- Standard written (compact) / column procedures to be learned for all four operations
- Efficient informal methods (expanded addition and subtraction, grid multiplication, division by chunking) and number lines are still used when appropriate. Develop these to larger numbers and decimals where appropriate.

N.B. Children must still be allowed access to practical resources to help visualise certain calculations, including those involving decimals

General Principles of Calculation

When faced with a calculation, children are able to decide which method is most appropriate and have strategies to check its accuracy.

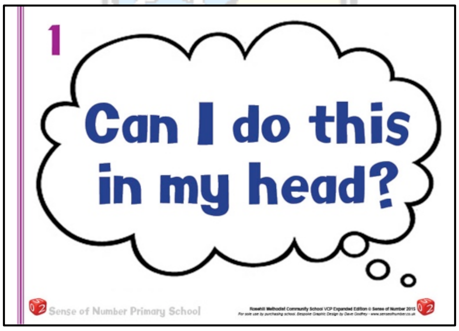
Whatever method is chosen (in any year group), it must still be underpinned by a secure and appropriate knowledge of number facts.

By the end of Year 5, children should:

- have a secure knowledge of number facts and a good understanding of the four operations in order to:
    - carry out calculations mentally when using one-digit and two-digit numbers
    - use particular strategies with larger numbers when appropriate
- use notes and jottings to record steps and part answers when using longer mental methods
- **have an efficient, reliable, compact written method of calculation for each operation that children can apply with confidence when undertaking calculations that they cannot carry out mentally;**

Children should always **look at the actual numbers (not the size of the numbers**) before attempting any calculation to determine whether or not they need to use a written method.

Therefore, the key question children should always ask themselves before attempting a calculation is: -



The Importance of Vocabulary in Calculation

It is vitally important that children are exposed to the relevant calculation vocabulary throughout their progression through the four operations.

Key Vocabulary: (to be used from Y1)

Addition: **Total & Sum** **Add**

E.g. 'The sum of 12 and 4 is 16', '12 add 4 equals 16'

'12 and 4 have a total of 16'

Subtraction: Difference

Subtract **(not 'take away' unless the strategy is take away / count back)**

E.g. 'The difference between 12 and 4 is 8',

'12 subtract 4 equals 8'

Multiplication: **Product** **Multiply**

E.g. 'The product of 12 and 4 is 48',

'12 multiplied by 4 equals 48'

Division: **Divisor & Quotient** **Divide**

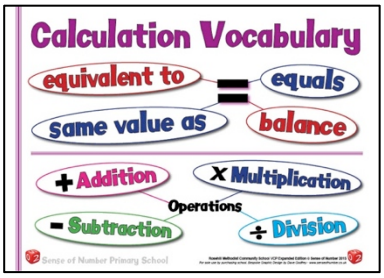
E.g. 'The quotient of 12 and 4 is 3',

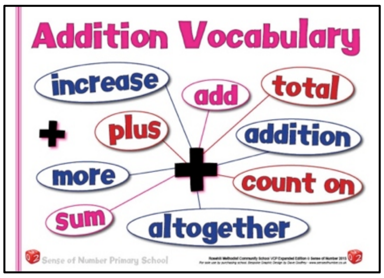
'12 divided by 4 equals 3'

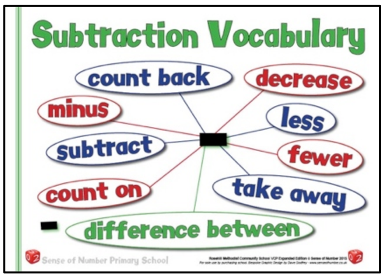
'When we divide 12 by 4, the divisor of 4 goes into 12 three times'

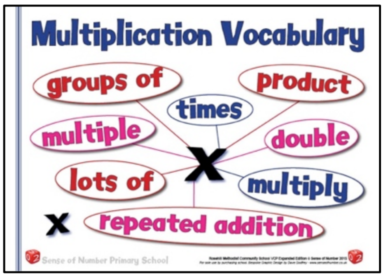
Additional Vocabulary:

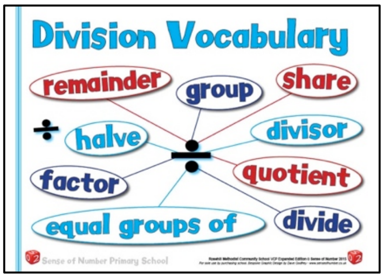
The VCP vocabulary posters (below) contain both the key and additional vocabulary children should be exposed to.





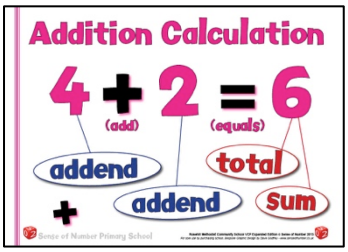


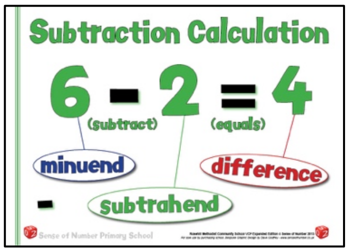


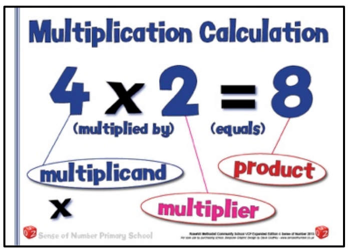


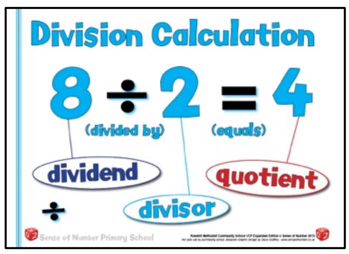
Conceptual Understanding

Using key vocabulary highlights some important conceptual understanding in calculation. For example, the answer in a subtraction calculation is called the difference. Therefore, whether we are counting back (taking away), or counting on, to work out a subtraction calculation, either way we are always finding the difference between two numbers.









Mental Methods of Calculation

Oral and mental work in mathematics is essential, particularly so in calculation.

Early practical, oral and mental work must lay the foundations by providing children with a good understanding of how the four operations build on efficient counting strategies and a secure knowledge of place value and number facts.

Later work must ensure that children recognise how the operations relate to one another and how the rules and laws of arithmetic are to be used and applied.

On-going oral and mental work provides practice and consolidation of these ideas. It must give children the opportunity to apply what they have learned to particular cases, exemplifying how the rules and laws work, and to general cases where children make decisions and choices for themselves.

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained and refined. A good knowledge of numbers or a **'sense' of number** is the product of structured practice and repetition. It requires an understanding of **number patterns and relationships** developed through **directed enquiry,** use of **models and images** and the application of **acquired number knowledge and skills**. Secure mental calculation requires the ability to:

- ***recall key number facts instantly*** – for example, all number bonds to 20, and doubles of all numbers up to double 20 **(Year 2)** and multiplication facts up to 12 × 12 **(Year 4);**
- **use taught strategies to work out the calculation** – for example, recognise that addition can be done in any order and use this to add mentally a one-digit number to a one-digit or two-digit number **(Year 1),** add two-digit numbers in different ways **(Year 2),** add and subtract numbers mentally with increasingly large numbers **(Year 5);**
- understand how the rules and laws of arithmetic are used and applied – for example to use commutativity in multiplication **(Year 2),** estimate the answer to a calculation and use inverse operations to check answers **(Years 3 & 4),** use their knowledge of the order of operations to carry out calculations involving the four operations **(Year 6).**

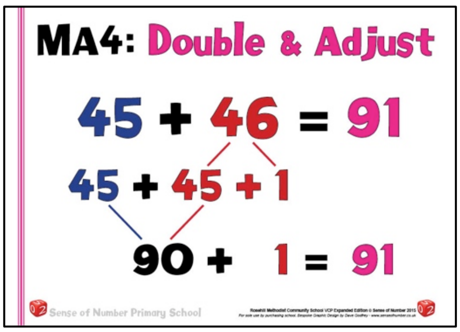
The first 'answer' that a child may give to a mental calculation question would be based on instant recall.

E.g. "What is 12 + 4?", "What is 12 x 4?", "What is 12 – 4?" or "What is 12÷ 4?" giving the immediate answers "16", "48", "8" or "3"

Other children would still work these calculations out mentally by counting on from 12 to 16, counting in 4s to 48, counting back in ones to 8 or counting up in 4s to 12.

From instant recall, children then develop a bank of mental calculation strategies for all four operations, in particular addition and multiplication.

These would be practised regularly until they become refined, where children will then start to see and use them as soon as they are faced with a calculation that can be done mentally.

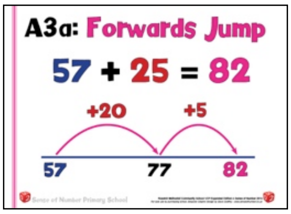


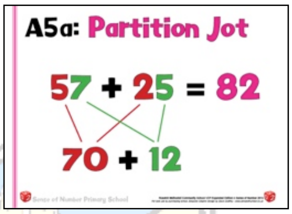
Informal Written Methods and Mental Jottings

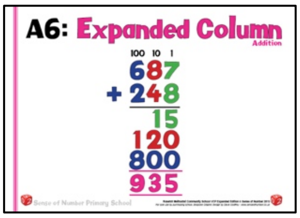
The ***New Curriculum for Mathematics*** sets out progression in written methods of calculation, which highlights the compact written methods for each of the four operations. It also places emphasis on the need to '*add and subtract numbers mentally*' (Years 2 & 3), mental arithmetic '*with increasingly large numbers*' (Years 4 & 5) and '*mental calculations with mixed operations and large numbers*' (Year 6). There is very little guidance, however, on the 'jottings' and informal methods that support mental calculation, and which provide the link between answering a calculation entirely mentally (without anything written down) and completing a formal written method with larger numbers.

This policy (especially in the progression of addition and multiplication) provides very clear guidance not only as to the development of formal written methods, but also the jottings, expanded and informal methods of calculation that embed a sense of number and understanding before column methods are taught. These extremely valuable strategies include:

Addition – number lines, partitioning, expanded methods

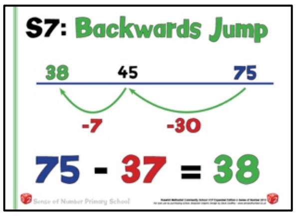


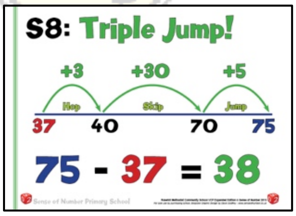


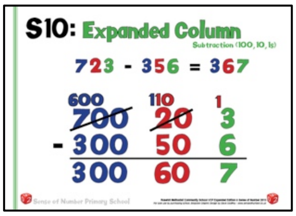


*(In addition to the 5 key mental strategies for addition - see '**Addition Progression')***

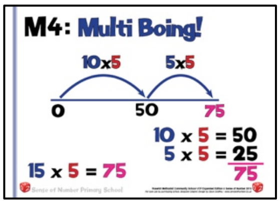
Subtraction – number lines (especially for counting on), expanded subtraction

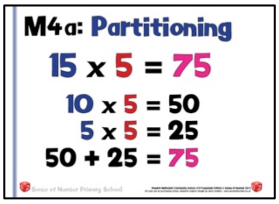


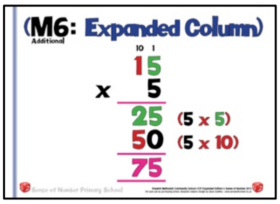


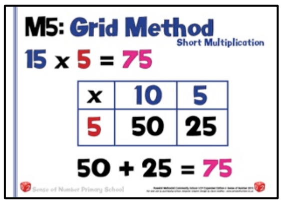


Multiplication – number lines, partitioning, expanded, grid method



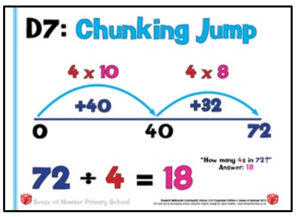


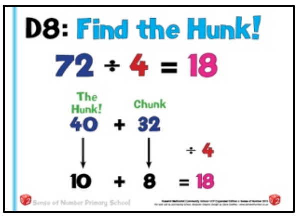


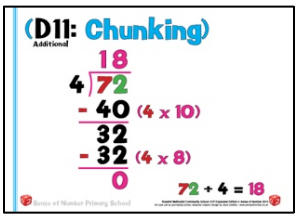


in addition to the key mental strategies for multiplication (see '***Multiplication Progression)***

Division – number lines, chunking (as a jotting), chunking (written method)







Formal (Column) Written Methods of Calculation

The aim is that by the end of **Year 5**, the great majority of children should be able to ***use an efficient written method for each operation with confidence and understanding with up to 4 digits***.

This guidance promotes the use of what are commonly known as 'standard' written methods – methods that are efficient and work for any calculation, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps.

Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. We want children to know that they have such a reliable, written method to which they can turn when the need arises.

In setting out these aims, the intention is that schools adopt greater consistency in their approach to calculation that all teachers understand and towards which they work.

There has been some confusion previously in the progression towards written methods and for too many children the staging posts along the way to the more compact method have instead become end points. While this may represent a significant achievement for some children, the great majority are entitled to learn how to use the most efficient methods.

The challenge for teachers is determining when their children should move on to a refinement in the method and become confident and more efficient at written calculation.

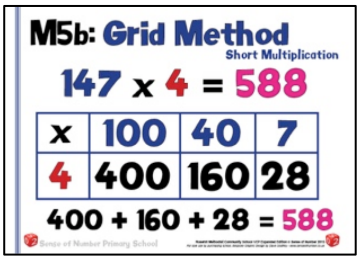
The incidence of children moving between schools and localities is very high in some parts of the country. Moving to a school where the written method of calculation is unfamiliar and does not relate to that used in the previous school can slow the progress a child makes in mathematics. There will be differences in practices and approaches, which can be beneficial to children. ***However, if the long-term aim is shared across all schools and if expectations are consistent then children's progress will be enhanced rather than limited***.

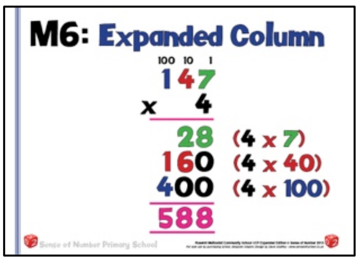
The entitlement to be taught how to use efficient written methods of calculation is set out clearly in the National Curriculum objectives. Children should be equipped to decide when it is best to use a mental or written method based on the knowledge that they are in control of this choice as they are able to carry out all methods with confidence.

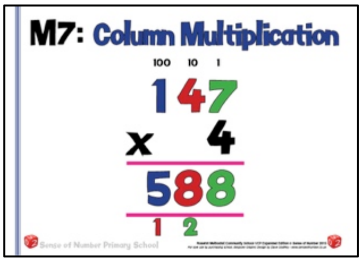
This policy does, however, clearly recognise that whilst children should be taught the efficient, formal written calculation strategies***, it is vital that they have exposure to models and images, and have a clear conceptual understanding of each operation and each strategy.***

The visual slides that feature below (in the separate progression documents) for all four operations have been taken from the Sense of Number Visual Calculations Policy.

They show, wherever possible, the different strategies for calculation exemplified with identical values. This allows children to compare different strategies and to ask key questions, such as, 'what's the same, what's different?'







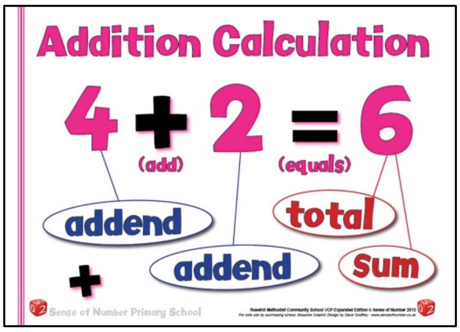
National Curriculum Objectives – Addition and Subtraction

Addition & Subtraction	1	2	3	4	5	6
Problem Solving	▪ solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = [ ] – 9.	▪ solve problems with addition and subtraction: ▪ ***using concrete objects and pictorial representations, including those involving numbers, quantities and measures ▪ applying their increasing knowledge of mental and written methods	▪ solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.	▪ solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.	▪ solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.	▪ solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why ▪ solve problems involving addition, subtraction, multiplication and division
Facts	▪ represent and use number bonds and related subtraction facts within 20	▪ recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100				
Understanding and Using Statements & Relationships	▪ read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs	▪ show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot ▪ recognise and use the inverse relationship between addition & subtraction and use this to check calculations and solve missing number problems.	▪ estimate the answer to a calculation and use inverse operations to check answers	▪ estimate and use inverse operations to check answers to a calculation	▪ use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy	▪ use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. ▪ use their knowledge of the order of operations to carry out calculations involving the four operations
Addition and Subtraction – Mental & Written Methods	▪ add and subtract one-digit and two-digit numbers to 20, including zero	▪ add and subtract numbers **using concrete objects, pictorial representations, and mentally, including: ▪ a two-digit number & ones ▪ a two-digit number & tens ▪ two two-digit numbers ▪ adding three one-digit numbers	▪ add and subtract numbers mentally, including: ▪ a three-digit number & ones ▪ a three-digit number & tens ▪ a three-digit number and hundreds ▪ add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	▪ add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	▪ add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) ▪ add and subtract numbers mentally with increasingly large numbers	▪ perform mental calculations, including with mixed operations and large numbers
Non Statutory Guidance	Pupils memorise and reason with number bonds to 10 and 20 in several forms (for example, 9 + 7 = 16; 16 – 7 = 9; 7 = 16 – 9). They should realise the effect of adding or subtracting zero. This establishes addition and subtraction as related operations. Pupils combine and increase numbers, counting forwards and backwards. They discuss and solve problems in familiar practical contexts, including using quantities. Problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.	Pupils extend their understanding of the language of addition and subtraction to include sum and difference. Pupils practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as using 3 + 7 = 10; 10 – 7 = 3 and 7 = 10 – 3 to calculate 30 + 70 = 100; 100 – 70 = 30 and 70 = 100 – 30. They check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, 5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5). This establishes commutativity and associativity of addition. Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers	Pupils practise solving varied addition and subtraction questions. For mental calculations with two-digit numbers, the answers could exceed 100. Pupils use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent (see Mathematics Appendix 1).	Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency (see English Appendix 1)	Pupils practise using the formal written methods of columnar addition and subtraction with increasingly large numbers to aid fluency (see Mathematics Appendix 1). They practise mental calculations with increasingly large numbers to aid fluency (for example, 12 462 – 2300 = 10 162).	Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division (see Mathematics Appendix 1). They undertake mental calculations with increasingly large numbers and more complex calculations. Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency. Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures. Pupils explore the order of operations using brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9. Common factors can be related to finding equivalent fractions.

National Curriculum Objectives – Multiplication and Division

Multiplication & Division	1	2	3	4	5	6
Problem Solving	▪ solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	▪ solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.	▪ solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.	▪ solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.	▪ solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes ▪ solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign ▪ solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.	▪ solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why ▪ solve problems involving addition, subtraction, multiplication and division ▪ use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.
Facts		▪ recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers	▪ recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables	▪ recall multiplication and division facts for multiplication tables up to 12 × 12	▪ establish whether a number up to 100 is prime and recall prime numbers up to 19	
Understanding and Using Statements & Relationships		▪ show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot		▪ use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers ▪ recognise and use factor pairs and commutativity in mental calculations	▪ identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers ▪ know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers ▪ recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})	▪ identify common factors, common multiples and prime numbers ▪ use their knowledge of the order of operations to carry out calculations involving the four operations
Multiplication and Division – Mental & Written Methods		▪ calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs	▪ write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods	▪ multiply two-digit and three-digit numbers by a one-digit number using formal written layout	▪ multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers ▪ multiply and divide numbers mentally drawing upon known facts ▪ divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context ▪ multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	▪ multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication ▪ divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context ▪ divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context ▪ perform mental calculations, including with mixed operations and large numbers

Addition Progression



The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

Children need to acquire **one efficient written method of calculation** for addition that they know they can rely on **when mental methods are not appropriate.**

To add successfully, children need to be able to:

- **recall all addition pairs to 9 + 9 and complements in 10;**
- **add mentally a series of one-digit numbers, such as 5 + 8 + 4;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

Mental Addition Strategies

There are 5 key mental strategies for addition, which need to be a regular and consistent part of the approach to calculation in all classes from Year 2 upwards.

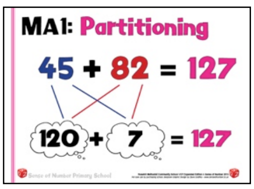
These strategies will be introduced individually when appropriate, and then be rehearsed and consolidated throughout the year until they are almost second nature.

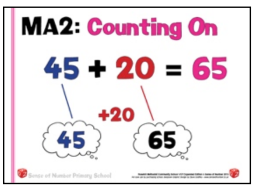
These strategies are **partitioning, counting on, round and adjust, double and adjust and using number bonds.** The first two strategies are also part of the written calculation policy (see pages 12-14) but can equally be developed as simple mental calculation strategies once children are skilled in using them as jottings.

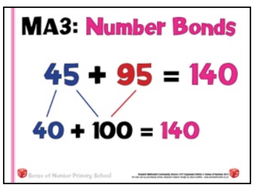
Using the acronym **RAPA CODA NUMBO**, children can be given weekly practice in choosing the most appropriate strategy whenever they are faced with a simple addition, usually of 2 or 3 digit numbers, but also spotting the opportunities (E.g. 3678 + 2997) when they can be used with larger numbers

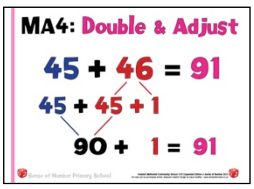
RA	**Round & Adjust**
PA	**Partitioning**
CO	**Counting On**
DA	**Double & Adjust**
NUMBO	**Number Bonds**

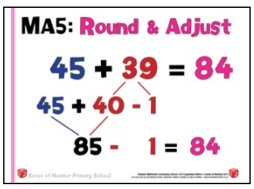
For example, using the number 45, we can look at the other number chosen, and decide on the most appropriate mental calculation strategy.











The 5 key strategies need to be linked to the key messages from pages 2 and 3 –

The choice as to whether a child will choose to use a mental method or a jotting will depend upon

a) the numbers chosen and

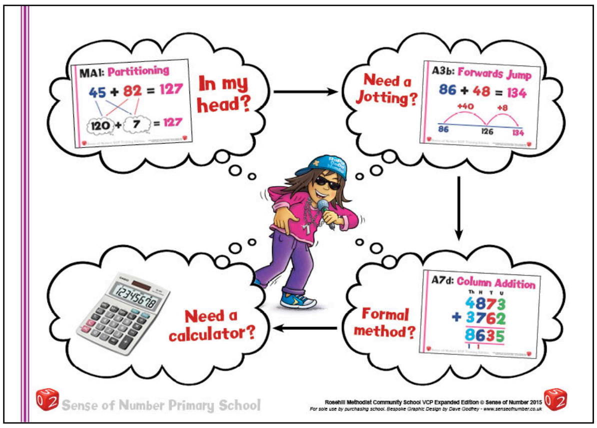
b) the level of maths that the child is working at.

For example, for **57 + 35**

a Year 2 child may use a long jotting or number line

a Year 3 child might jot down a quick partition jotting,

a Year 4 child could simply partition and add mentally.

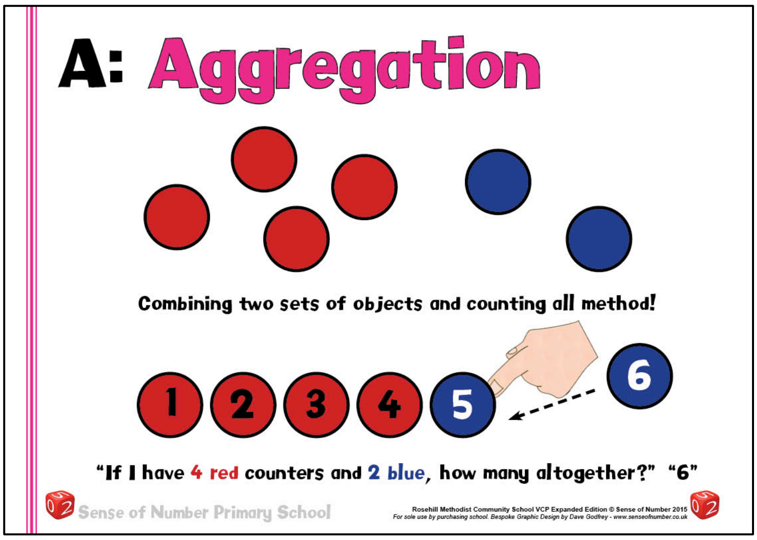
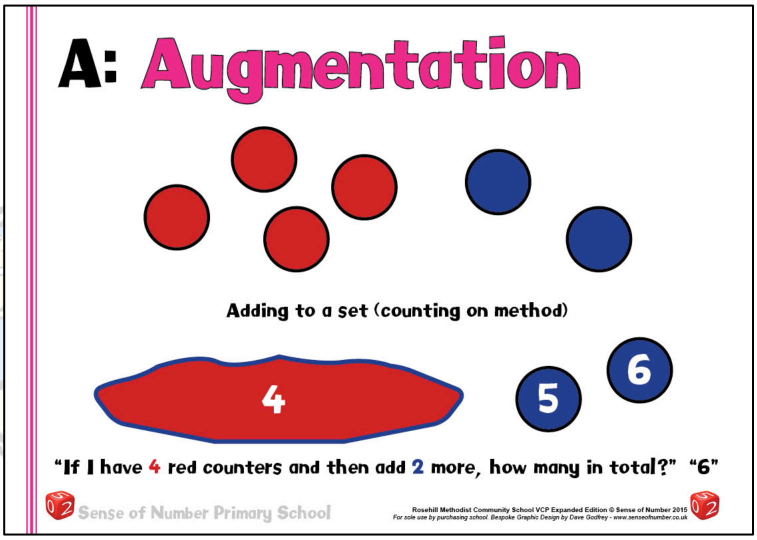


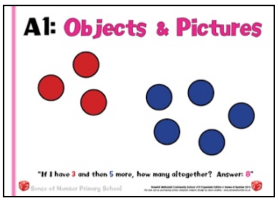
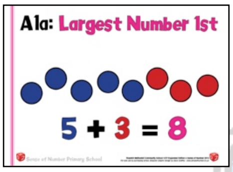
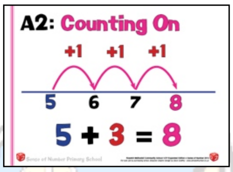
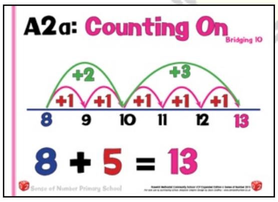
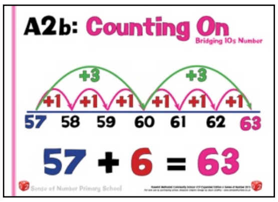
As a strategy develops, a child will begin to recognise the instances when it would be appropriate: -

E.g. 27 +9, 434 + 197, 7.6 + 1.9 and 5.86 + 3.97 can all be calculated very quickly by using the **Round & Adjust** strategy.

Below you can see the progression of each strategy through the year groups, with some appropriate examples of numbers, which may be used for each strategy.

MA	MA1: Partitioning 45 + 82 = 127 120 + 7 = 127	MA2: Counting On 45 + 20 = 65 +20 45 65		MA3: Number Bonds 45 + 95 = 140 40 + 100 = 140	MA4: Double & Adjust 45 + 46 = 91 45 + 45 + 1 90 + 1 = 91	MA5: Round & Adjust 45 + 39 = 84 45 + 40 - 1 85 - 1 = 84
Y1		MA2a: Counting On 12 + 5 = 17 +5 12 17	MA2b: Counting On 57 + 10 = 67 +10 57 67	MA3: Number Bonds	MA4: Double & Adjust 5 + 6 = 11 5 + 5 + 1 10 + 1 = 11	MA5: Round & Adjust 45 + 9 = 54 45 + 10 - 1 = 55 - 1 = 54
Y2	MA1: Partitioning 43 + 21 = 64 60 + 4 = 64	MA2a: Counting On 78 + 7 = 85 +7 78 85	MA2b: Counting On 58 + 40 = 98 +40 58 98	MA3: Number Bonds 3 + 4 + 7 = 14 10 4	MA4: Double & Adjust 7 + 8 = 15 7 + 7 + 1 14 + 1 = 15	MA5: Round & Adjust 45 + 19 = 64 45 + 20 - 1 65 - 1 = 64
Y3	MA1: Partitioning 57 + 25 = 82 70 + 12 = 82	MA2a: Counting On 85 + 50 = 135 +50 85 135	MA2b: Counting On 534 + 300 = 834 +300 534 834	MA3: Number Bonds 43 + 9 + 7 + 21 = 80 50 30	MA4: Double & Adjust 16 + 17 = 33 16 + 16 + 1 32 + 1 = 33	MA5: Round & Adjust 45 + 97 = 142 45 + 100 - 3 145 - 3 = 142
Y4	MA1: Partitioning 648 + 231 = 879 800 + 70 + 9 = 879	MA2a: Counting On 784 + 60 = 844 +60 784 844	MA2b: Counting On 4837 + 3000 = 8347 +3000 4837 7837	MA3: Number Bonds 42 + 16 + 28 + 54 = 140 70 70	MA4: Double & Adjust 37 + 38 = 75 37 + 37 + 1 74 + 1 = 75	MA5: Round & Adjust 345 + 298 = 643 345 + 300 - 2 645 - 2 = 643
Y5	MA1: Partitioning 576 + 258 = 834 700 + 120 + 14 = 834	MA2a: Counting On 837 + 500 = 1337 +500 837 1337	MA2b: Counting On 7583 + 5000 = 12583 +5000 7583 12583	MA3: Number Bonds £4.56 + £3.27 + £1.44 = £9.27 £6.00 £3.27	MA4: Double & Adjust 125 + 127 = 252 125 + 125 + 2 250 + 2 = 252	MA5: Round & Adjust 4645 + 1996 = 6641 4645 + 2000 - 4 6645 - 4 = 6641
Y6	MA1: Partitioning 4.73 + 2.21 = 6.94 6 + 0.9 + 0.04 = 6.94	MA2a: Counting On 43,826 + 30,000 = 73,826 +30,000 43,826 73,826	MA2b: Counting On 5,763,947 + 4,000,000 = 9,763,947 +4,000,000 5,763,947 9,763,947	MA3: Number Bonds 24.25 + 31.63 + 21.75 = 77.63 46 31.63	MA4: Double & Adjust 4.5 + 4.7 = 9.2 4.5 + 4.5 + 0.2 9 + 0.2 = 9.2	MA5: Round & Adjust 45.2 + 49.9 = 95.1 45.2 + 50 - 0.1 95.2 - 0.1 = 95.1

Models	Addition
Aggregation (Combining sets and counting all)	
Augmentation (Adding to an existing set - counting on)	

Stage 1	Finding a Total and the Empty Number Line	Alternative Method: Counting on Mentally or as a jotting
FS/Y1	Initially, children need to represent addition using a range of different resources, and understand that a total can be found by counting out the first number, counting out the second number then counting how many there are altogether. **3 + 5 = 8**	
		3 (held in head) then use fingers to count on 5 ("3… 4,5,6,7,8)
	This will quickly develop into placing the largest number first, either as a pictorial / visual method or by using a number line. **5 + 3 = 8**	
	 	5 (held in head) then count on 3 ("5 … 6, 7, **8**")
Y1/2	Steps in addition can be recorded on a number line. The steps often bridge through 10. **8 + 5 = 13**	
		8 (held in head) then count on 5 ("8 … 9, 10, 11, 12, **13**")
	The next step is to bridge through a multiple of 10.	
		57 (held in head) then count on 6 ("57 … 58,59,60,61,62,63")
	The number line becomes a key image for demonstrating how to keep one number whole, whilst partitioning the other number. Teach the children firstly to add the tens then the ones individually (43 + 24 = 43 + 10 + 10 + 1 + 1 + 1 + 1) before progressing to counting on in tens and ones (43 + 20 + 4)	This method will be a jotting approach, and may look like the following examples: - 43 + 24 43 + 20 = 63 63 + 4 = 67

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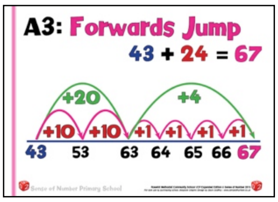
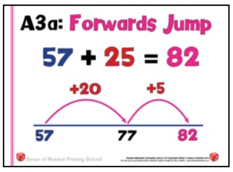
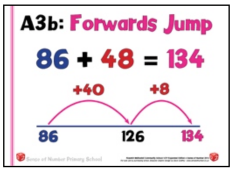
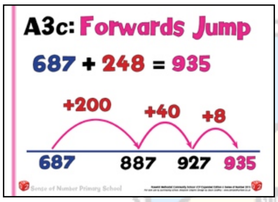
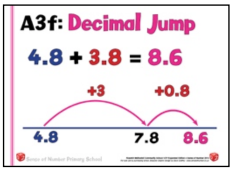
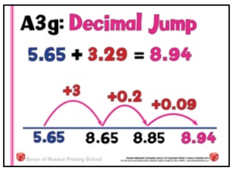
<td>
</td>
<td>Or
43 + 20 + 4 = 67</td>
</tr>
<tr>
<td></td>
<td>Develop to crossing the 10s, then the 100s boundary
57 + 25 = 82 86 + 48 = 134</td>
<td></td>
</tr>
<tr>
<td></td>
<td>

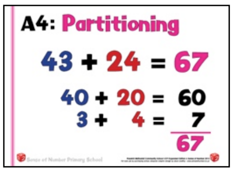

</td>
<td>57 + 25
57 + 20 = 77
77 + 5 = 82
57 + 20 + 5 = 82

86 + 48
86 + 40 = 126
126 + 8 = 134
86 + 40 + 8 = 1</td>
</tr>
<tr>
<td>Y3/4</td>
<td>For some children, this method can still be used for 3 digit calculations</td>
<td>Number lines support children's thinking if they find partitioning / column addition difficult, as it simply involves counting on in 100s, 10s & 1s.</td>
</tr>
<tr>
<td></td>
<td>
</td>
<td>687 + 248
687 + 200 = 887
887 + 40 = 927
927 + 8 = 935
Or
687 + 200 + 40 + 8 = 935</td>
</tr>
<tr>
<td>Y5/6</td>
<td>In Years 5 and 6, if necessary, children can return to this method to support their understanding of decimal calculation</td>
<td></td>
</tr>
<tr>
<td></td>
<td>

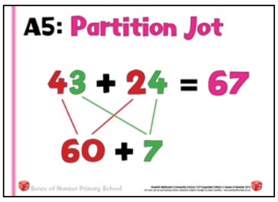

</td>
<td>4.8 + 3.8
4.8 + 3 = 7.8
7.8 + 0.8 = 8.6
Or
4.8 + 3 + 0.8 = 8.6</td>
</tr>
</table>

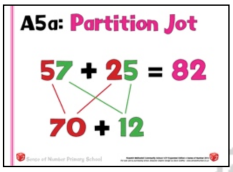
Hopefully, with the above calculation, many children would mentally Round & Adjust (4.8 + 4 – 0.2 = 8.6)

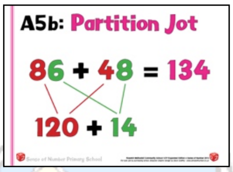
<table>
<tr><th>Stage 2</th><th>Partition Jot</th><th>Alternative Method: Traditional Partitioning</th></tr>
<tr><td>Y2/3</td><td>Traditionally, partitioning has been presented using the method on the right. Although this does support place value and the use of arrow cards, it is very laborious, so it is suggested that adopting the 'partition jot' method will improve speed and consistency for mental to written (or written to mental) progression</td><td rowspan="2">Record steps in addition using partition, initially as a jotting: -
43 + 24 = 40 + 20 + 3 + 4 =
60 + 7 = 67
Or, preferably



</td></tr>
<tr><td></td><td>As soon as possible, refine this method to a much quicker and clearer 'Partition Jot' approach



</td></tr>
<tr><td></td><td>As before, develop these methods, especially Partition Jot, towards crossing the 10s and then 100s.</td><td></td></tr>
<tr><td></td><td>




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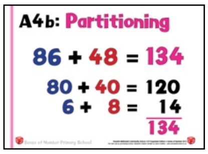
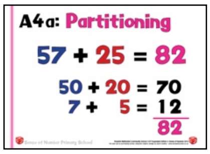
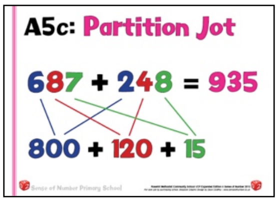
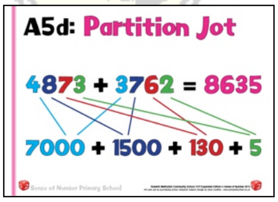




</td></tr>
<tr><td></td><td>This method will soon become the recognised jotting to support the teaching of partitioning. It can be easily extended to 3 and even 4 digit numbers when appropriate.</td><td>For certain children, the traditional partitioning method can still be used for 3 digit numbers, but is probably too laborious for 4 digit numbers.</td></tr>
<tr><td>Y3/4</td><td>




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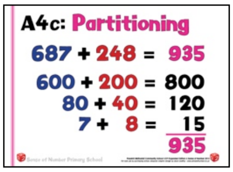
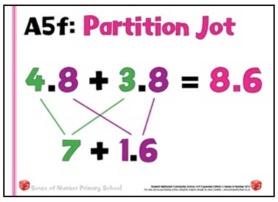
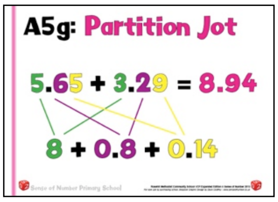
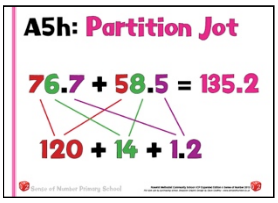
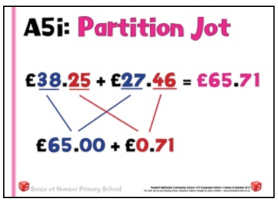


</td></tr>
<tr><td></td><td>Partition jot is also extremely effective as a quicker alternative to column addition for decimals.</td><td>Some simple decimal calculations can also be completed this way.</td></tr>
<tr><td>Y5/6</td><td>




</td><td></td></tr>
<tr><td></td><td>For children with higher-level decimal place value skills, partition jot can be used with more complex decimal calculations or money.</td><td></td></tr>
<tr><td></td><td>




</td><td></td></tr>
</table>

Stage 3 — Expanded Method in Columns

Y3

Column methods of addition are introduced in Year 3, but it is crucial that they still see mental calculation as their first principle, especially for 2 digit numbers.

Column methods should only be used for more difficult calculations, usually with 3 digit numbers that cross the Thousands boundary or most calculations involving 4 digit numbers and above.

N.B. Even when dealing with bigger numbers / decimals, children should still look for the opportunity to calculate mentally (E.g. 4675 + 1998)

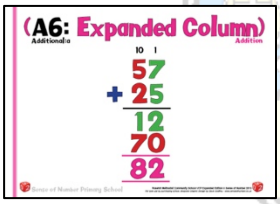
2 digit examples are used below simply to introduce column methods to the children. Most children would continue to answer these calculations mentally or using a simple jotting.

Using the column, children need to learn the principle of adding the ones first rather than the tens.

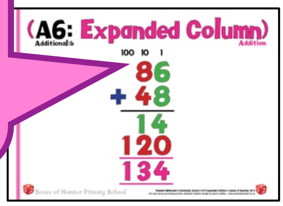
The 'expanded' method is a very effective introduction to column addition. It continues to use the partitioning strategy that the children are already familiar with, but begins to set out calculations vertically. It is particularly helpful for automatically 'dealing' with the 'carry' digit

Y3/4

A. Single 'carry' in units

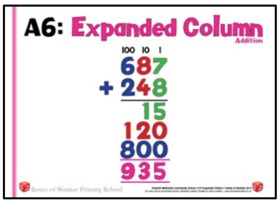


B. 'Carry' in units and tens



'Eighty plus forty equals one hundred and twenty, because 'eight plus four equals twelve.

Once this method is understood, it can quickly be adapted to using with three digit numbers. It is rarely used for 4 digits and beyond as it becomes too unwieldy.



The time spent on practising the expanded method will depend on security of number facts recall and understanding of place value.

Once the children have had enough experience in using expanded addition, and have also used practical resources (Base 10 / place value counters) to model exchanging in columns, they can be taken on to standard, 'traditional' column addition.

Stage 4 — Column Method

Y3/4

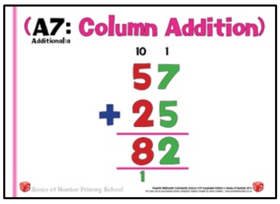
As with the expanded method, begin with 2 digit numbers, simply to demonstrate the method, before moving to 3 digit numbers.

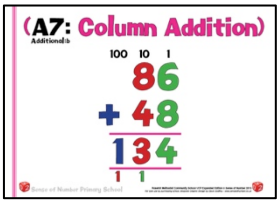
Make it very clear to the children that they are still expected to deal with all 2 digit (and many 3 digit) calculations mentally (or with a jotting), and that the column method is designed for numbers that are too difficult to access using these ways. The column procedure **is not** intended for use with 2 digit numbers.

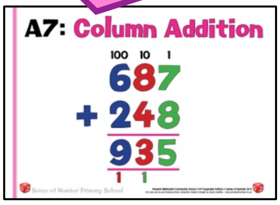
'Carry' ones then ones and tens

Use the words 'carry ten' and 'carry hundred', not 'carry one'

Record carry digits below the line.

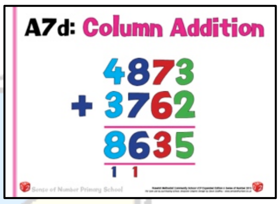






Y4

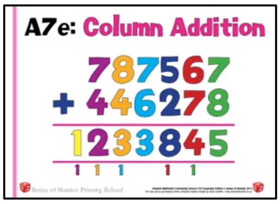
Once confident, use with 4 digit numbers **(Year 4).**

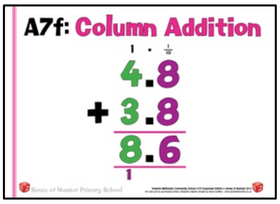


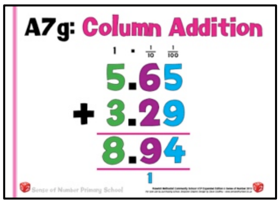
Y5/6

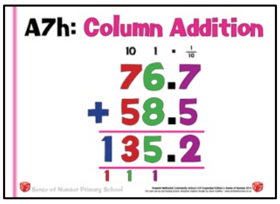
Extend to 5/6 digit calculations then decimal calculations **(Year 5)**

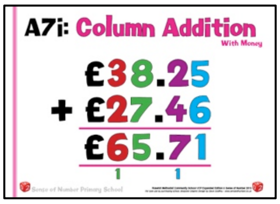
If children make repeated errors at any stage, they can return to the expanded method or an earlier jotting.



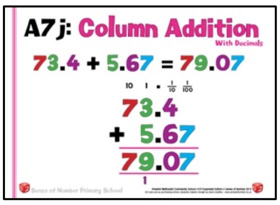




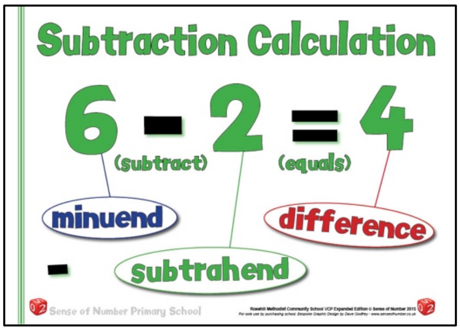




The key skill in upper Key Stage 2 that needs to be developed is the laying out of the column method for calculations with decimals in different places.



Subtraction Progression



The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

To subtract successfully, children need to be able to:

- **recall all addition and subtraction facts to 20;**
- **subtract multiples of 10 (such as 160 – 70) using the related subtraction fact (e.g. 16 – 7), and their knowledge of place value;**
- **partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Children need to acquire **one efficient written method of calculation for subtraction**, which they know they can rely on **when mental methods are not appropriate.**

NOTE: They should look at the actual numbers each time they see a calculation and decide whether or not their favoured method is most appropriate (e.g. If there are zeroes in a calculation such as 206 -198) then the 'counting on' approach may well be the best method in that particular instance).

Therefore, when subtracting, whether mental or written, children will mainly choose between two main strategies to find the difference between two numbers: -

Counting Back (Taking away) ***Counting On***

When should we count back and when should we count on?

This will alter depending on the calculation (see below), but often the following rules apply;

If the numbers are far apart, or there isn't much to subtract (278 – 24) then count back.

If the numbers are close together (206 – 188), then count up

In many cases, either strategy would be suitable, depending on preference (743 – 476)

<table>
<tr><th>Models</th><th>Subtraction</th></tr>
<tr><td>

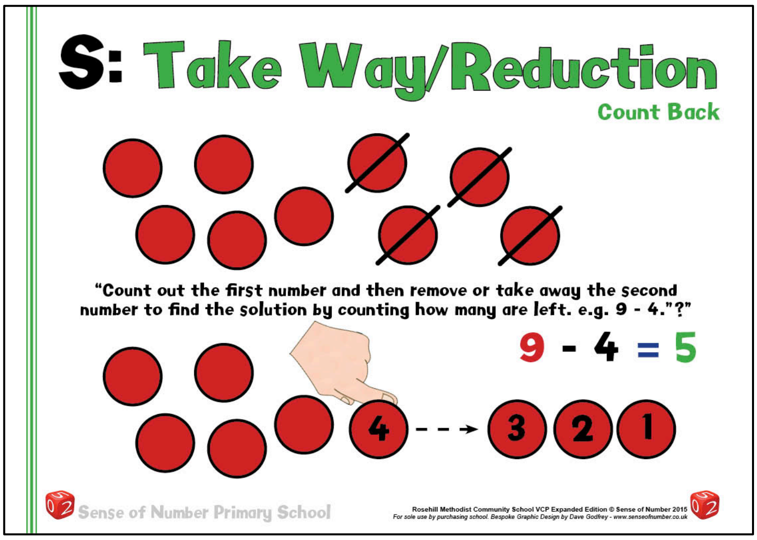
Removing items from a set:

A: Take Away

B: Reduction

(Count Back Images)

</td><td>



</td></tr>
<tr><td></td><td>

Take Away: Samir has 12 cakes and Nihal takes 5 cakes. How many cakes does Samir now have?

Reduction: The shoes originally cost £12, but have been reduced in the sale by £5. How much do they now cost?

</td></tr>
<tr><td>

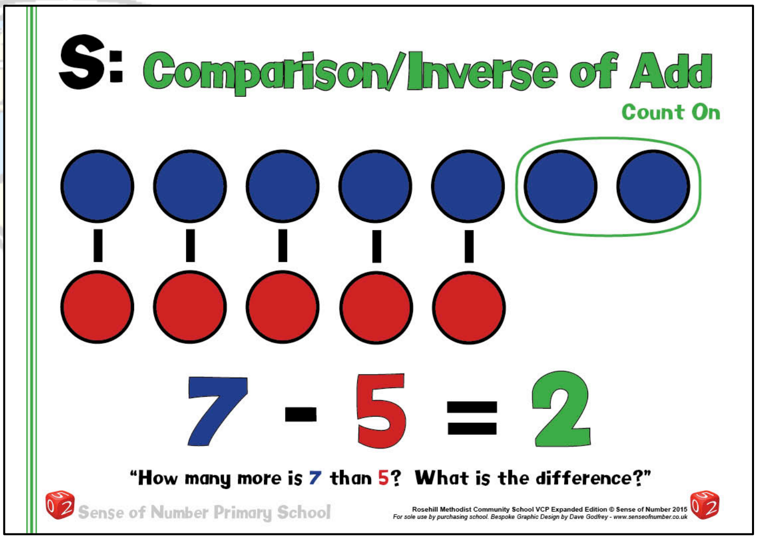
Comparing two sets:

A: Comparison

B: Inverse of Addition

(Counting Up/On Images)

</td><td>

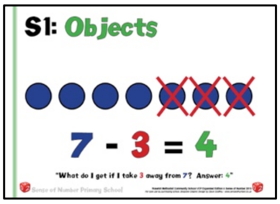
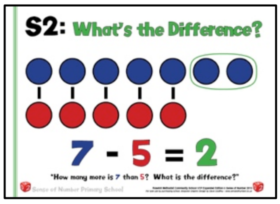


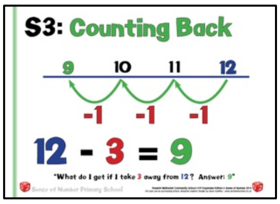
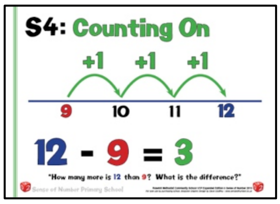
</td></tr>
<tr><td></td><td>

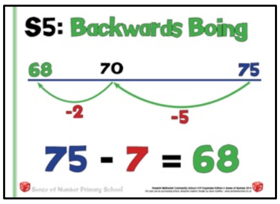
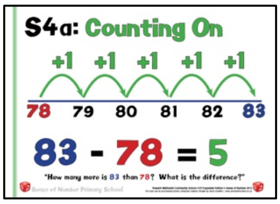
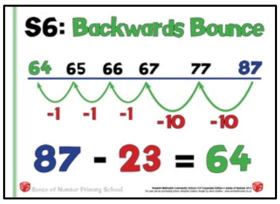
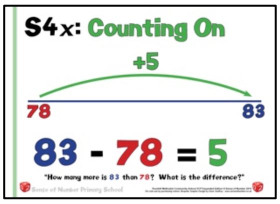
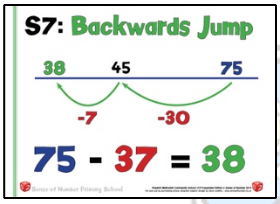
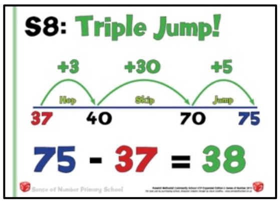
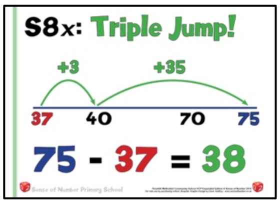
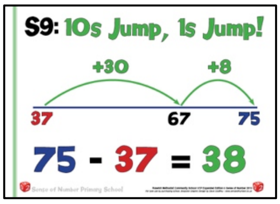
Comparision: Samir has 12 cakes and Nihal has 5 cakes. How many more cakes does Samir have than Nihal?

Inverse of Addition: The shoes cost £12, but I've only got £5. How much more money will I need in order to buy the shoes? (5 + ? = 12)

</td></tr>
</table>

INTRO	Subtraction by counting back (or taking away)	Subtraction by counting up (or complementary addition)
FS/Y1	Early subtraction in EYFS will primarily be concerned with '***taking away'***, and will be modelled using a wide range of models and resources.	
		
	This will continue in Year 1, using resources and images (including the desktop number track / line) to practise taking away practically, and then counting back on demarcated number lines.	In Year 1, it is also vital that children understand the concept of subtraction as ***'finding a difference'*** and realise that <u>**any**</u> subtraction can be answered in 2 different ways, either by counting up or counting back. Again, this needs to be modelled and consolidated regularly using a wide range of resources, especially multilink towers, counters and Numicon.
		

Stage 1	Using the empty number line	
	Subtraction by counting back (or taking away)	**Subtraction by counting up (or complementary addition)**
	The empty number line helps to record or explain the steps in mental subtraction. It is an ideal model for **counting back** and **bridging ten**, as the steps can be shown clearly. It can also show **counting up** from the smaller to the larger number to **find the difference.**	
Y1	The steps often bridge through a multiple of 10. **12 – 3 = 9**	Small differences can be found by counting up **12 – 9 = 3**
		
Y2/3	This is developed into crossing any multiple of 10 boundary. **75 – 7 = 68**	For 2 (or 3) digit numbers close together, count up **83 – 78 = 5** First, count in ones

		
	For 2 digit numbers, count back in 10s and 1s **87 – 23 = 64**	Then, use number facts to count in a single jump
		
	Then subtract tens and units in single jumps (87 – 20 – 3)	Continue to spot small differences with 3 digit numbers **(403 – 397 = 6)**
	Some numbers (75 – 37) can be subtracted just as quickly either way.	
	Either count back 30 then count back 7	Or count up from smaller to the larger number, initially with a 'triple jump' strategy of jumping to the next 10, then multiples of 10, then to the target number.
		
		This can also be done in 2 jumps.
		
		Some children prefer to jump in tens and ones, which is an equally valid strategy, as it links to the mental skill of *'counting up from any number in tens'*
		

Stage 2	Expanded Method & Number Lines (continued)	
	Subtraction by counting back Expanded Method	**Subtraction by counting up Number Lines (continued)**

In Year 3, according to the New Curriculum, children are expected to be able to use both jottings <u>and</u> written column methods to deal with 3 digit subtractions.

This is only guidance, however – as long as children leave Year 6 able to access all four operations using formal methods, schools can make their own decisions as to when these are introduced.

It is very important that they have had regular opportunities to use the number line 'counting up' approach first (right hand column below) so that they already have a secure method that is almost their first principle for most 2 and 3 digit subtractions.

This means that once they have been introduced to the column method they have an alternative approach that is often preferable, depending upon the numbers involved.

The number line method also gives those children who can't remember or successfully apply the column method an approach that will work with any numbers (even 4 digit numbers and decimals) if needed.

It is advisable to spend at least the first two terms in Year 3 focusing upon the number line / counting up approach through regular practice, then introducing column method in the 3rd term as an alternative, or even waiting until Year 4 to introduce columns.

Ideally, whenever columns are introduced, the expanded method should be practised in depth (potentially up until 4 digit calculations are introduced)

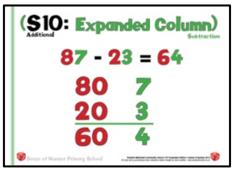
Y3/4

The expanded method of subtraction is an excellent way to introduce the column approach as it maintains the place value and is much easier to model practically with place value equipment such as Base 10 or place value counters

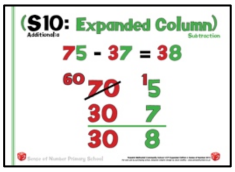
Introduce the expanded method with 2 digit numbers, but only to explain the process. Column methods are very rarely needed for 2 digit calculations.

Partition both numbers into tens and ones, firstly with no exchange then exchanging from tens to the ones.

87 – 23

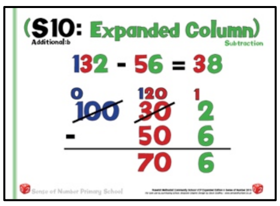


75 – 37

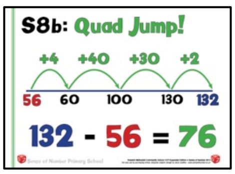


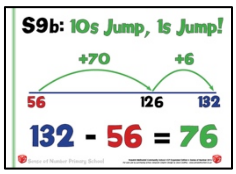
Develop into exchanging from hundreds to tens and tens to ones.

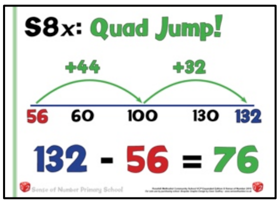
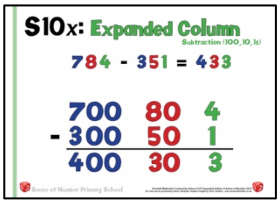
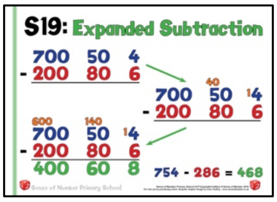
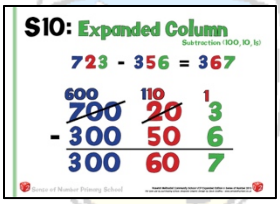
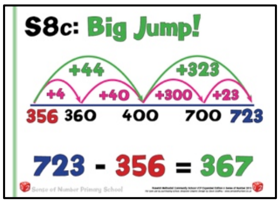
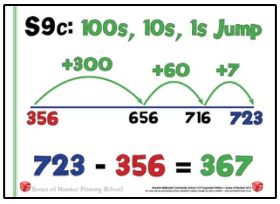
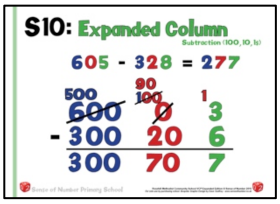
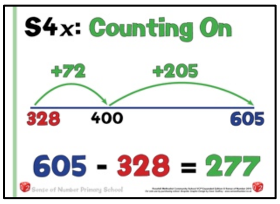
132 – 56

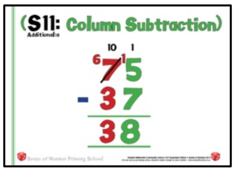
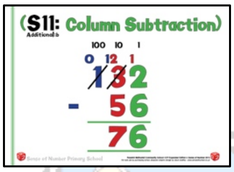
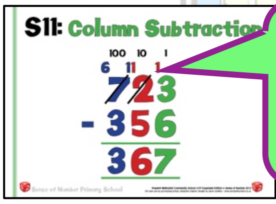
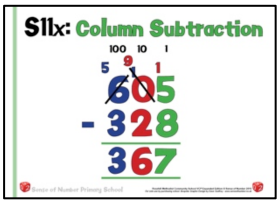
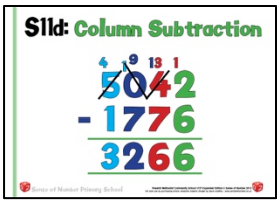
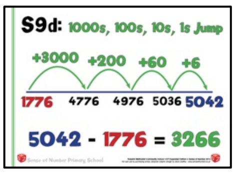
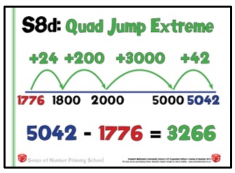


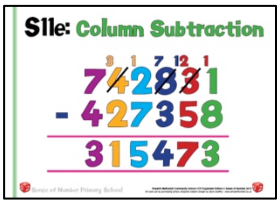
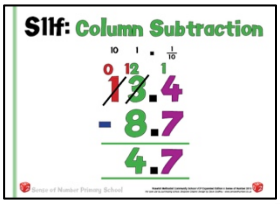
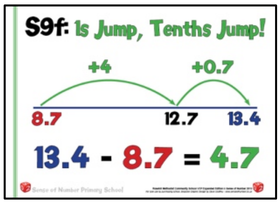
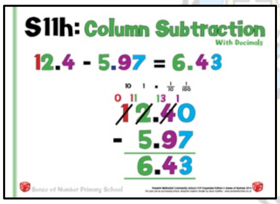
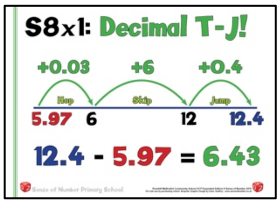
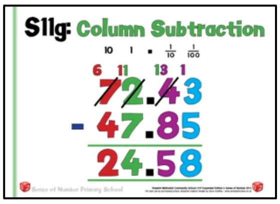
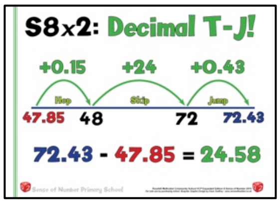
The number line method is equally as effective when crossing the hundreds boundary, either by the triple / quad jump strategy or by counting in tens then ones.



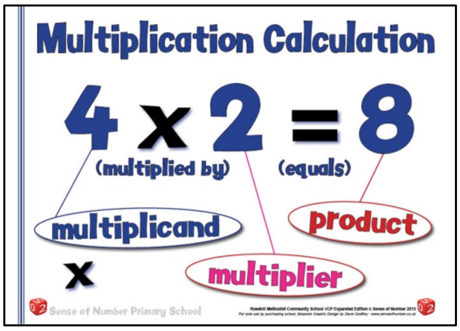


		The 'quad jump' can be completed by many children in fewer steps, either a triple or **double** jump. 
A	Take the method into three digit numbers. Subtract the ones, then the tens, then the hundreds. Demonstrate without exchanging first. 784 – 351	***For examples without exchanging, the number line method takes considerably longer than mental partitioning or expanded.***
	 	
B	Move towards exchanging from hundreds to tens and tens to ones, in two stages if necessary 723 – 356	The example below shows 2 alternatives, for children who need different levels of support from the image.
	  	
	For examples where exchanging is needed, then the number line method is equally as efficient, and is often easier to complete	As before, many children prefer to count in hundreds, then tens, then ones.
		
C	Use some examples which include the use of zeros e.g. 605 – 328.	***For numbers containing zeros, counting up is often the most reliable method.***
	 →	
	Continue to use expanded subtraction until both number facts and place value are considered to be very secure!	

Stage 3	Standard Column Method (decomposition)	
	Subtraction by counting back **Standard Method**	**Subtraction by counting up** **Number Lines (continued)**
Mainly **Y4+**	**Decomposition relies on secure understanding of the expanded method, and simply displays the same numbers in a contracted form.** As with expanded method, and using practical resources such as place value counters to support the teaching, children in Years 3 or 4 (depending when the school introduces the column procedure) will quickly move from decomposition via 2-digit number 'starter' examples to 2 / 3 digit and then 3 digit columns. **75 – 37** **132 – 56**  	
	723 – 356 	***Continue to refer to digits by their actual value, not their digit value, when explaining a calculation. E.g. One hundred and twenty subtract fifty.***
	Again, use examples containing zeros, remembering that it may be easier to count on with these numbers (see Stage 2) **605 – 328** 	**It is even possible, for children who find column method very difficult to remember, or who regularly make the same mistakes, to use the number line method for 4 digit numbers, using either of the approaches.**
Y4	From late Y4 onwards, move onto examples using 4 digit (or larger) numbers and then onto decimal calculations. **5042 – 1776** 	**5042 – 1776**  

Y5/6	In Years 5 & 6 apply to any 'big number' examples.	
		
	Both methods can be used with decimals, although the counting up method becomes less efficient and reliable when calculating with more than two decimal places.	
	13.4 – 8.7	13.4 – 8.7
		
	12.4 – 5.97	12.4 – 5.97
		
	72.43 – 47.85	
		

Multiplication Progression



The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

These notes show the stages in building up to using an efficient method for

- two-digit by one-digit multiplication by the end of Year 3,
- three-digit by one-digit multiplication by the end of Year 4,
- four-digit by one-digit multiplication **and** two/three-digit by two-digit multiplication by the end of Year 5
- three/four-digit by two-digit multiplication **and** multiplying 1-digit numbers with up to 2 decimal places by whole numbers by the end of Year 6.

To multiply successfully, children need to be able to:

- **recall all multiplication facts to 12 × 12;**
- **partition numbers into multiples of one hundred, ten and one;**
- **work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;**
- **similarly apply their knowledge to simple decimal multiplications such as 0.7 x 5, 0.7 x 0.5, 7 x 0.05, 0.7 x 50 using the related fact 7 × 5 and their knowledge of place value;**
- **add two or more single-digit numbers mentally;**
- **add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;**
- **add combinations of whole numbers using the column method (see above).**

Note:

Children need to acquire **one efficient written method of calculation for multiplication, which** they know they can rely on **when mental methods are not appropriate.**

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

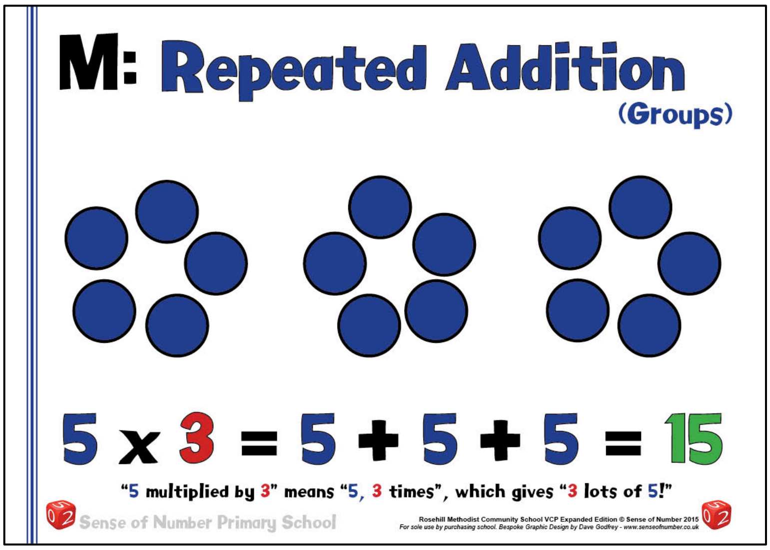
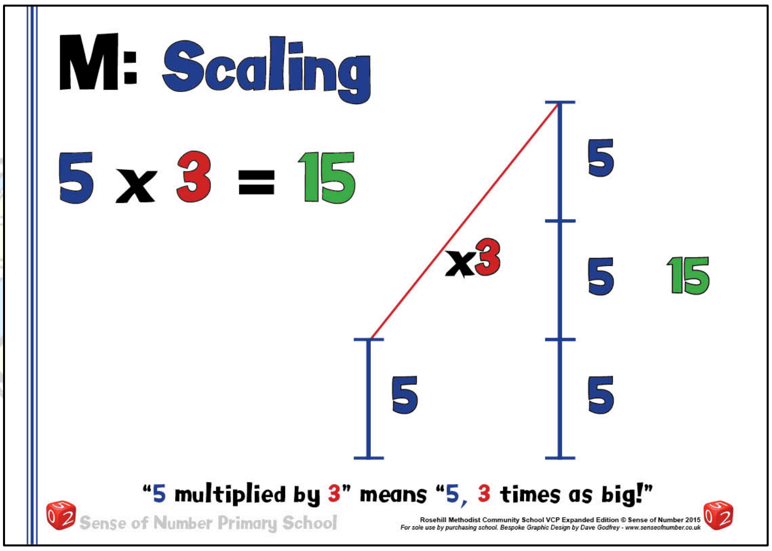
These mental methods are often more efficient than written methods when multiplying.

Use partitioning and grid methods until number facts and place value are secure

For a calculation such as 25 x 24, a quicker method would be 'there are four 25s in 100 so 25 x 24 = 100 x 6 = 600

When multiplying a 3 / 4 digit x 2-digit number the standard method is usually the most efficient

At all stages, use known facts to find other facts. E.g. Find 7 x 8 by using 5 x 8 (40) and 2 x 8 (16)

Models	Multiplication
Repeated Addition	
Scaling	

Mental Multiplication

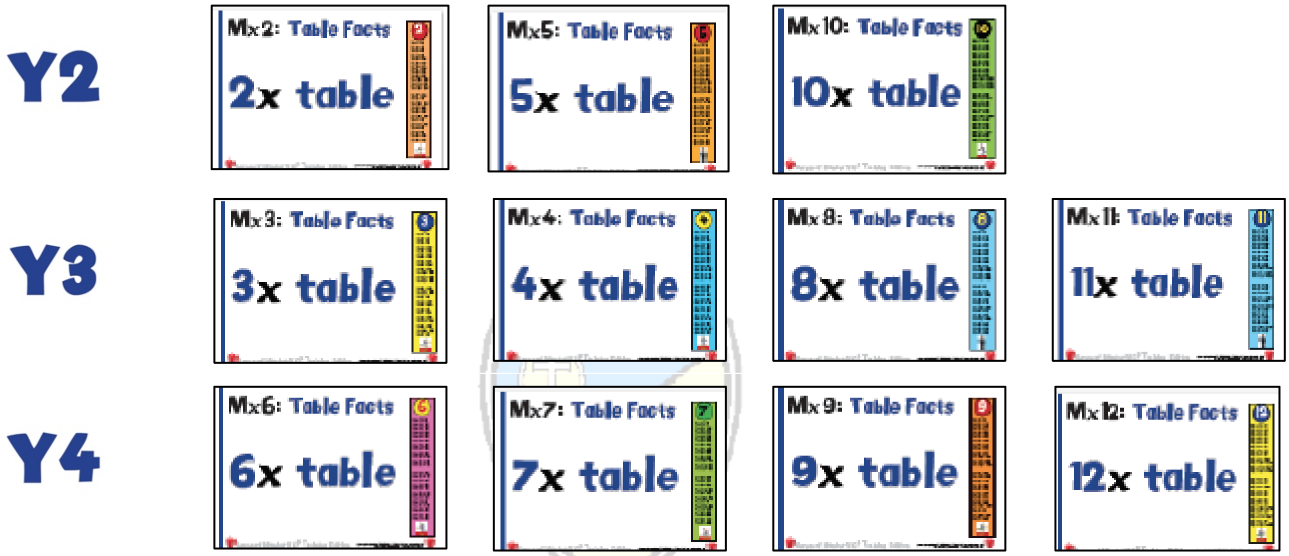
In a similar way to addition, multiplication has a range of mental strategies that need to be developed both before and then alongside written methods (both informal and formal).

Tables Facts

In Key Stage 2, however, before any written methods can be securely understood, children need to have a bank of multiplication tables facts at their disposal, which can be recalled instantly.

The learning of tables facts does begin with counting up in different steps, but by the end of Year 4 it is expected that most children can instantly recall all facts up to 12 x 12.

The progression in facts is as follows (11's moved into Y3 as it is a much easier table to recall): -



Once the children have established a bank of facts, they are ready to be introduced to jottings and eventually written methods.

Doubles & Halves

The other facts that children need to know by heart are doubles and halves. These are no longer mentioned explicitly within the National Curriculum, making it even more crucial that they are part of a school's mental calculation policy. If children haven't learned to recall simple doubles instantly, and haven't been taught strategies for mental doubling, then they cannot access many of the mental calculation strategies for multiplication (E.g. Double the 4 times table to get the 8 times table. Double again for the 16 times table etc.).

As a general guidance, children should know the following doubles: -

Year Group	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Doubles and Halves	All doubles and halves from double 1 to double 10 / half of 2 to half of 20	All doubles and halves from double 1 to double 20 / half of 2 to half of 40 (E.g.double 17=34, half of 28 = 14)	Doubles of all numbers to 100 with units digits 5 or less, and corresponding halves (E.g. Double 43, double 72, half of 46) Reinforce doubles & halves of all multiples of 10 & 100 (E.g. double 800, half of 140)	Addition doubles of numbers 1 to 100 (E.g. 38 + 38, 76 + 76) and their corresponding halves Revise doubles of multiples of 10 and 100 and corresponding halves	Doubles and halves of decimals to 10 – 1 d.p. (E.g. double 3.4, half of 5.6)	Doubles and halves of decimals to 100 – 2 d.p. (E.g. double 18.45, half of 6.48)

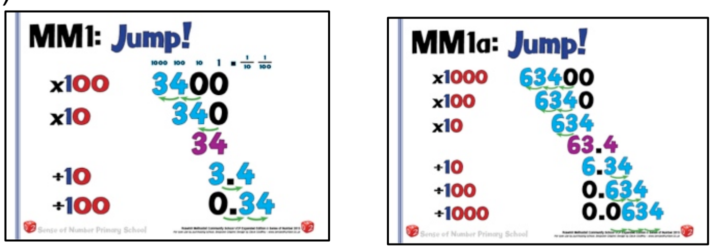
Before certain doubles / halves can be recalled, children can use a simple jotting to help them record their steps towards working out a double / half

As mentioned, though, there are also several mental calculation strategies that need to be taught so that children can continue to begin any calculation with the question 'Can I do it in my head?' The majority of these strategies are usually taught in Years 4 – 6, but there is no reason why some of them cannot be taught earlier as part of the basic rules of mathematics.

Multiplying by 10 / 100 / 1000

The first strategy is usually part of the Year 5 & 6 teaching programme for decimals, namely that digits move to the left when multiplying by 10, 100 or 1000, and to the right when dividing.

This also secures place value by emphasising that the decimal point doesn't ever move, and that the digits move around the decimal point (not the other way round, as so many adults were taught at school).

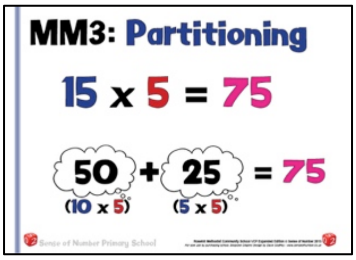


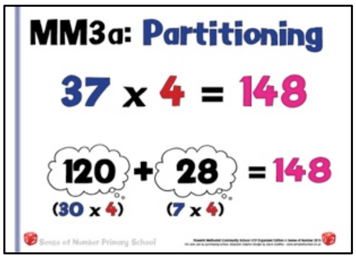
It would be equally beneficial to teach a simplified version of this strategy in KS1 / Lower KS2, encouraging children to move digits into a new column, rather than simply 'adding zeroes' when multiplying by 10/100.

The following 3 strategies can be explicitly linked to 3 of the strategies in mental addition (**Partitioning**, **Round & Adjust** and **Number Bonds**)

Partitioning is an equally valuable strategy for multiplication, and can be quickly developed from a jotting to a method completed entirely mentally. It is clearly linked to the grid method of multiplication, but should also be taught as a 'partition jot' so that children, by the end of Year 4, have become skilled in mentally partitioning 2 and 3 digit numbers when multiplying (with jottings when needed).

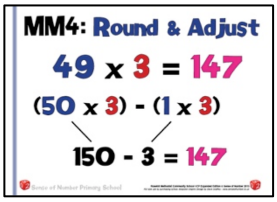
By the time they leave Year 6 they should be able to mentally partition most simple 2 & 3 digit, and also decimal multiplications.

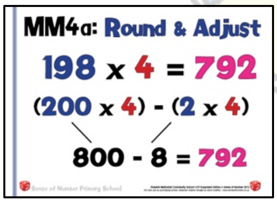


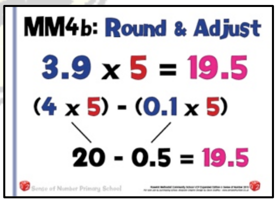


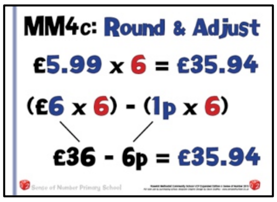
Round & Adjust is also a high quality mental strategy for multiplication, especially when dealing with money problems in upper KS2. Once children are totally secure with rounding and adjusting in addition, they can be shown how the strategy extends into multiplication, where they round then adust by the multiplier.

E.g. For 39 x 6 round to 40 x 6 (240) then adjust by 1 x 6 (6) to give a product of 240 – 6 = 234.





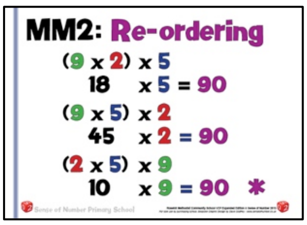


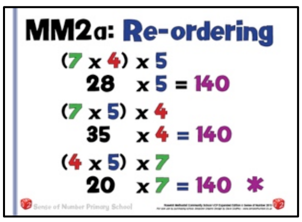


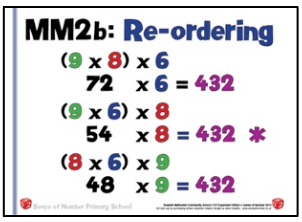
Y4 **Y4/5** **Y5** **Y5/6**

Re-ordering is similar to **Number Bonds** in that the numbers are calculated in a different order. I.e. The children look at the numbers that need to be multiplied, and, using commutativity, rearrange them so that the calculation is easier.

The asterisked calculation in each of the examples below is probably the easiest / most efficient rearrangement of the numbers.

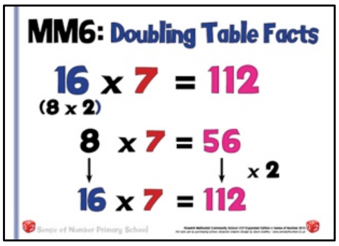




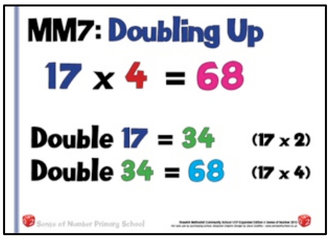


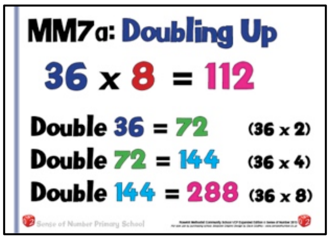
Doubling strategies are probably the most crucial of the mental strategies for multiplication, as they can make difficult long multiplication calculations considerably simpler.

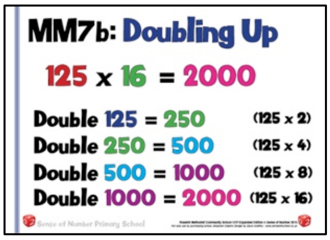
Initially, children are taught to double one table to find another (E.g..doubling the 3s to get the 6s) This can then be applied to any table: -



Doubling Up enables multiples of **4, 8** and **16** onwards to be calculated by constant doubling: -

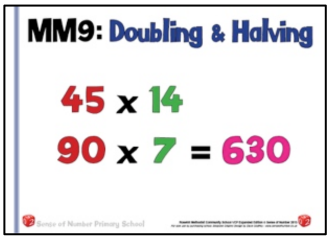


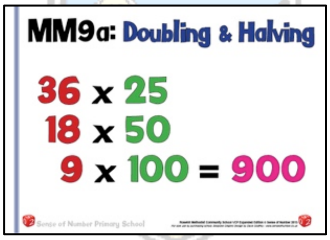


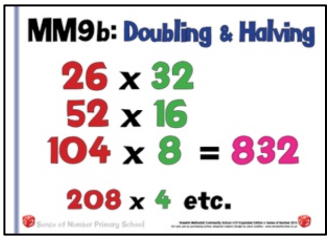


Doubling & Halving is probably the best strategy available for simplifying a calculation.

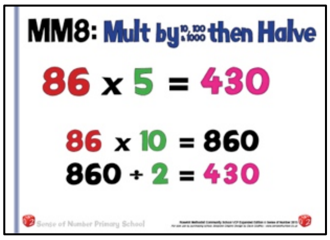
Follow the general rule that if you double one number within a multiplication, and halve the other number, then the product stays the same.

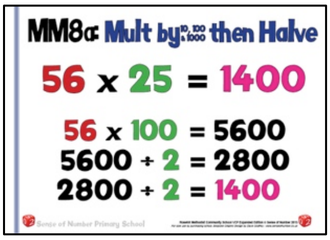




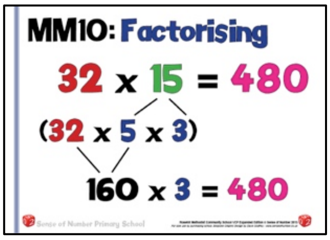


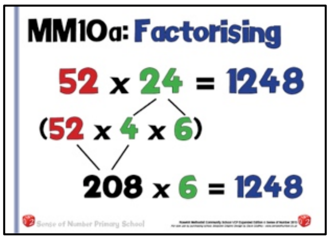
Multiplying by 10 / 100 / 1000 then halving. The final doubling / halving strategy works on the principle that multiplying by 10 / 100 is straightforward, and this can enable you to easily multiply by 5, 50 or 25.





Factorising The only remaining mental strategy, which again can simplify a calculation, is factorising. Multiplying a 2-digit number by 36, for example, may be easier if multiplying by a factor pair of 36 (x6 then x6, or x9 then x4, even x12 then x3)





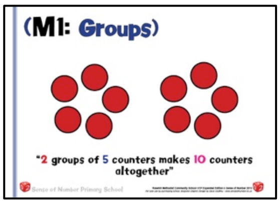
Written Multiplication

Stage 1	Number Lines, Arrays & Mental Methods
FS	In Early Years, children are introduced to grouping, and are given regular opportunities to put objects into groups of 2, 3, 4, 5 and 10. They also stand in different sized groups, and use the term ‘pairs’ to represent groups of 2. Using resources such as Base 10 apparatus, Numicon, multi-link or an abacus, children visualise counting in ones, twos, fives and tens, saying the multiples as they count the pieces. E.g. Saying ’10, 20, 30’ or ‘Ten, 2 tens, 3 tens’ whilst counting Base 10 pieces

Y1

Begin by introducing the concept of multiplication as repeated addition.

Children will make and draw objects in groups (again using resources such as Numicon, counters and multi-link), giving the product by counting up in 2s, 5s, 10s and beyond, and writing the multiplication statement.



Extend into making multiplication statements for 3s and 4s, using resources (especially real life equipment such as cups, cakes, sweets etc.)

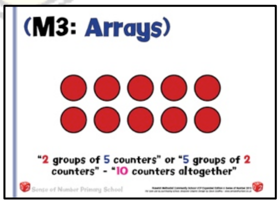
Make sure from the start that all children say the multiplication fact the correct way round, using the word ‘multiply’ more often than the word ‘times’.

For the example above, there are 5 counters in 2 groups, showing **5 multiplied by 2 (5x2), not** 2 times 5. It is the **‘5’** which is being scaled up / made bigger / **multiplied**.

‘5 multiplied by 2’ shows ‘2 groups of 5’ or ‘Two fives’

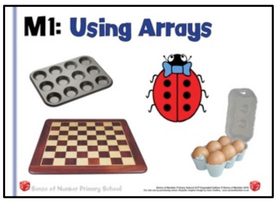
Develop the use of the array to show linked facts (commutativity).

Emphasise that all multiplications can be worked out either way. (2 x 5 = 5 x 2 = 10)



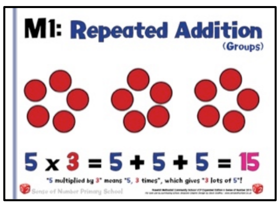
Y2

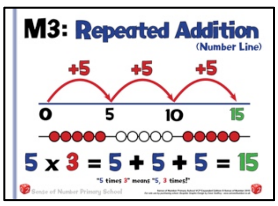
Build on children’s understanding that multiplication is repeated addition, using arrays and number lines to support the thinking. Explore arrays in real life.

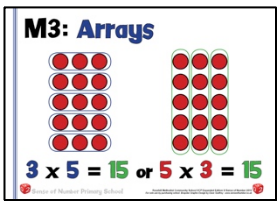


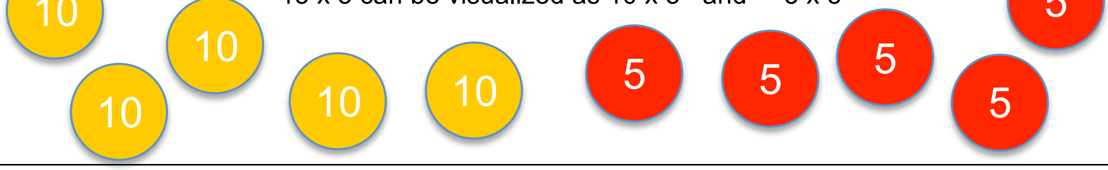
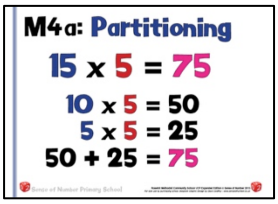
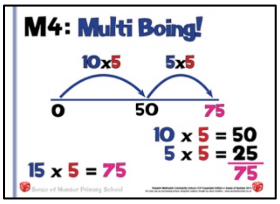
Start to emphasise commutativity, e.g. that 5 x 3 = 3 x 5

Continue to emphasise multiplication the correct way round.
E.g. 5 x 3 = 5 + 5 + 5
5 multiplied by 3 = 15







Y3	Extend the above methods to include the 3, 4, 6 and 8 times tables. Continue to model calculations, where appropriate, with resources such as Numicon, Place Value Counters or the Slavonic abacus, counting quickly in different steps and placing / moving the resource. Extend the use of resources to 2 digit x 1 digit calculations so that children can visualize what the calculation looks like. 15 x 5 can be visualized as 10 x 5 and 5 x 5 
	Then begin to partition using **jottings and number lines.**   ***Each of these methods can be used in the future if children find expanded or standard methods difficult.***
	Extend the methods above to calculations which give products greater than 100.

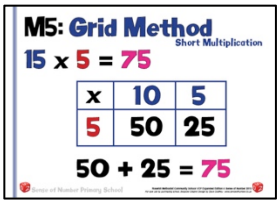
NB. – Use of 'grid' method within the New Curriculum

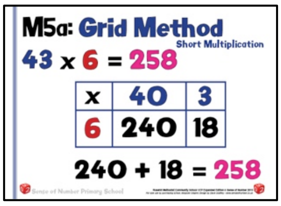
In the New Curriculum, the Grid Method is not exemplified as a written method for multiplication.

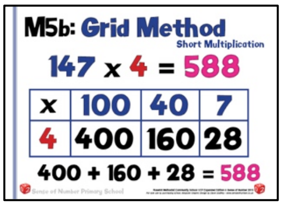
The only methods highlighted and specifically mentioned are column procedures.

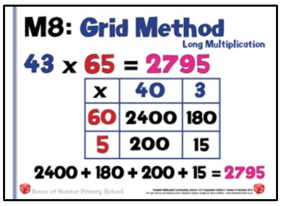
Most schools in the UK, however, have effectively built up the use of the grid method over the past 15 years, and it is generally accepted as the most appropriate method for simple 2 and 3 digit x single digit calculations, as well as 2 digit x 2 digit calculations. It develops clear understanding of place value as well as being an efficient method, and is especially useful in Years 4 and 5.

Consequently, grid method is a key element of this policy, but, to align with the New Curriculum, could be classed as a mental 'jotting' as it builds on partitioning, and is also the key mental multiplication method used by children in KS2 (see page 29 – multiplication partitioning.

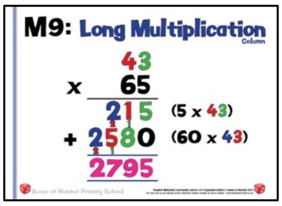




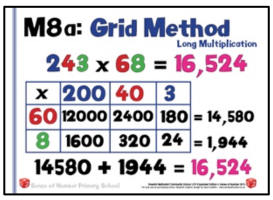


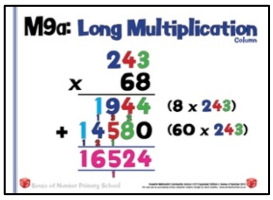


Column procedures still retain some element of place value, but, particularly for long multiplication, tend to rely on memorising a 'method', and can lead to many children making errors with the method (which order to multiply the digits, when to 'add the zero', dealing with the 'carry' digits' etc.) rather than the actual calculation.

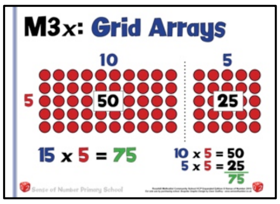
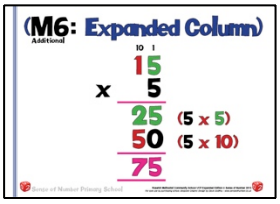
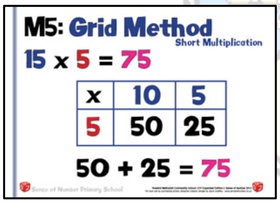
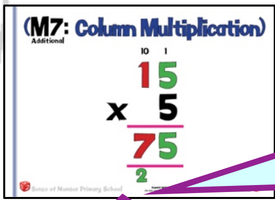
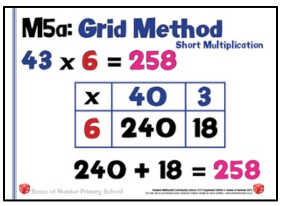
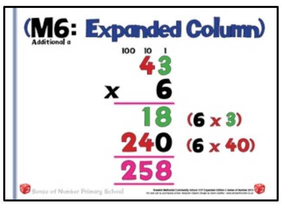
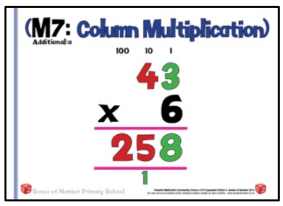


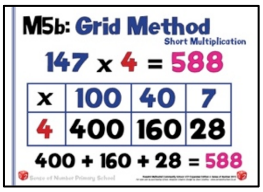
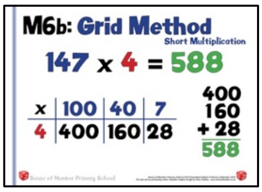
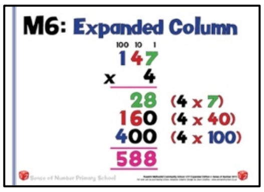
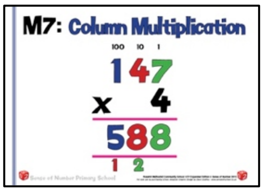
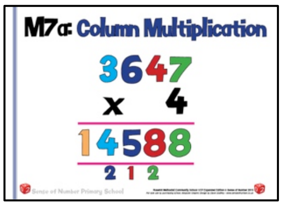
Once the calculations become more unwieldy (4 digit x 1 digit or 3 / 4 digit x 2 digit) then grid method begins to lose its effectiveness, as there are too many zeroes and part products to deal with. At this stage column procedures are far easier, and, once learned, can be applied much quicker. Grid methods can still be used by some pupils who find columns difficult to remember, and who regularly make errors, but children should be encouraged to move towards columns for more complex calculations





Stage 2 Written Methods - Short Multiplication

	Grid Multiplication (Mental 'Jotting')	Column multiplication (Expanded method into standard)
	The grid method of multiplication is a simple, alternative way of recording the jottings shown previously. *If necessary (for some children) it can initially be taught using an array to show the actual product.*	The expanded method links the grid method to the standard method. It still relies on partitioning the tens and units, but sets out the products vertically. Children will use the expanded method until they can securely use and explain the standard method.
		
Y3	It is recommended that the grid method is used as the main method within Year 3. It clearly maintains place value, and helps children to visualise and understand the calculation better. 	At some point within the year (preferably the 3rd term), the column method can be introduced, and children given the choice of using either grid or standard. Some schools may delay the introduction of column method until Year 4  ***Place the 'carry' digit below the line*** ***When setting out calculations vertically, begin with the ones first (as with addition and subtraction).***
Y4	Continue to use both grid and column methods in Year 4 for more difficult 2 digit x 1 digit calculations, extending the use of the grid method into mental partitioning for those children who can use the method this way. At this point, the expanded method can still be used when necessary (to help 'bridge' grid with column), but children should be encouraged to use their favoured method (grid or column) whenever possible.	
		 
	For 3 digit x 1 digit calcualtions, both grid and standard methods are efficient. Continue to use the grid method to aid place value and mental arithmetic. Develop column method for speed, and to make the transition to long multiplication easier.	

	If both methods are taught consistently then children in Year 4 will have a clear choice of 2 secure methods, and will be able to develop both accuracy and speed in multiplication. ***If children find it difficult to add the 'part products' then set them out vertically (or encourage column method)***
	   
Y5	For a 4 digit x 1 digit calculation, the column method, once mastered, is quicker and less prone to error. The grid method may continue to be the main method used by children who find it difficult to remember the column procedure, or children who need the visual link to place value. 

Stage 3 — Long Multiplication (TU x TU)

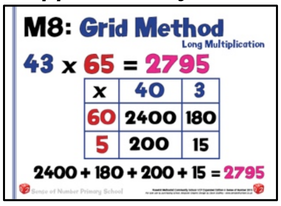
	Grid Multiplication	Column multiplication (Expanded method into standard)

Y5

Grid Multiplication

Extend the grid method to TU × TU, asking children to estimate first so that they have a general idea of the answer.

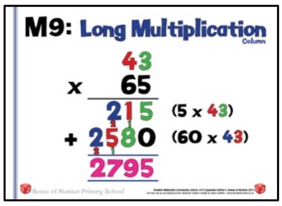
(43 × 65 is approximately 40 × 70 = 2800.)



As mentioned earlier, the grid method is often the 'choice' of many children in Years 5 and 6, due to its ease in both procedure and understanding / place value and is the method that they will mainly use for simple long multiplication calculations.

Column multiplication

Children should only use the 'standard' column method of long multiplication if they can regularly get the correct answer using this method.



There is no 'rule' regarding the position of the 'carry'digits. Each choice has advantages and complications.

Either carry the digits mentally or have your own favoured position for these digits.

Y6

Grid Multiplication

For 3 digit x 2 digit calculations, grid method is quite inefficient, and has much scope for error due to the number of 'part-products' that need to be added.

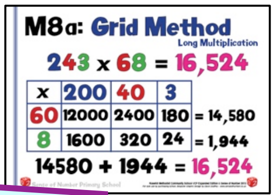
Use this method when you find the standard method to be unreliable or difficult to remember.

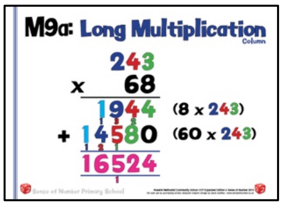
Column multiplication

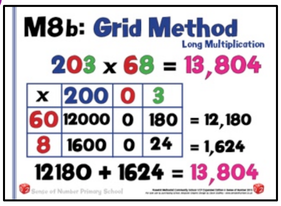
Most children, at this point, should be encouraged to choose the standard method. For 3 digit x 2 digit calculations it is especially efficient, and less prone to errors when mastered. **Although they may find the grid method easier to apply, it is much slower / less efficient.**

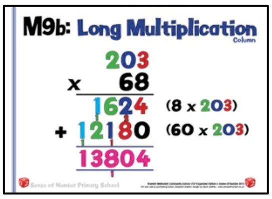
Again, estimate first:
243 × 68 is approximately 200 × 70 = 14000.

Add these numbers for the overall product



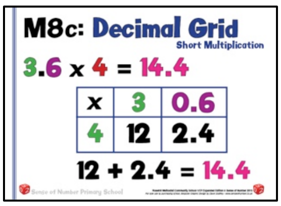


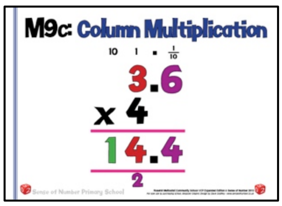




Many children will find the use of Grid method as an efficient method for multiplying decimals.

Extend the use of standard method into the use of decimals.





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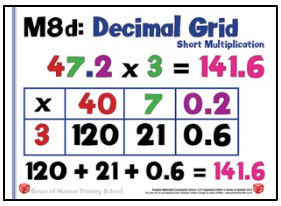

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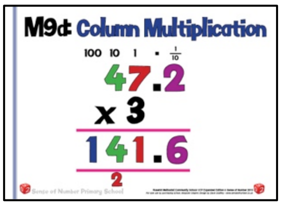

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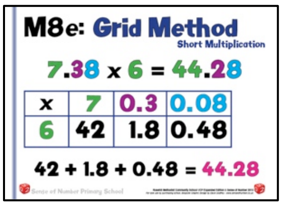

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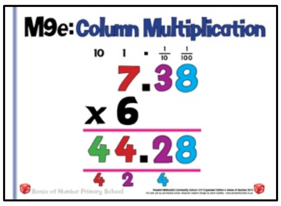

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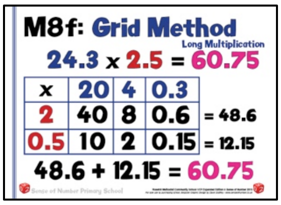

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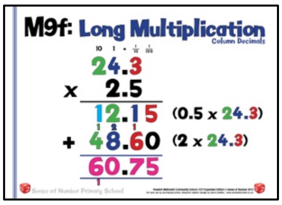

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<tr>
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<td></td>
<td>By this time children meet 4 digits by 2 digits, the only efficient method is the standard method for Long Multiplication.</td>
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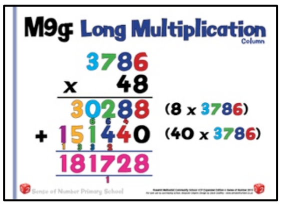

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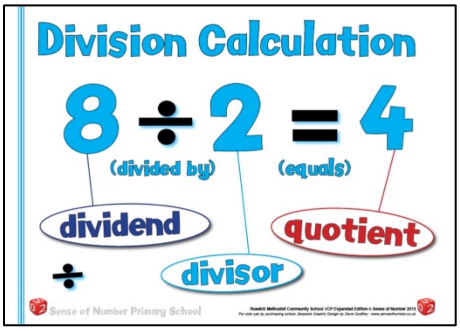
Division Progression



The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

These notes show the stages in building up to long division through Years 3 to 6 – first using short division 2 digits ÷ 1 digit, extending to 3 / 4 digits ÷ 1 digit, then long division 4 / 5 digits ÷ 2 digits.

To divide successfully in their heads, children need to be able to:

- **understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;**
- **partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;**
- **recall multiplication and division facts to 12 × 12, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;**
- **know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;**
- **understand and use multiplication and division as inverse operations.**

Children need to acquire **one efficient written method of calculation for division**, which they know they can rely on **when mental methods are not appropriate.**

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out expanded and standard written methods of division successful, children also need to be able to:

- **visualise how to calculate the quotient by visualising repeated addition;**
- **estimate how many times one number divides into another – for example, approximately how many sixes there are in 99, or how many 23s there are in 100;**
- **multiply a two-digit number by a single-digit number mentally;**
- **understand and use the relationship between single digit multiplication, and multiplying by a multiple of 10. (e.g. 4 x 7 = 28 so 4 x 70 = 280 or 40 x 7 = 280 or 4 x 700 = 2800.)**
- **subtract numbers using the column method (if using NNS 'chunking')**

For example, without a clear understanding that 72 can be partitioned into 60 and 12, 40 and 32 or 30 and 42 (as well as 70 and 2), it would be difficult to divide 72 by 6, 4 or 3 using the 'chunking' method.

72 ÷ 6 'chunks' into 60 and 12

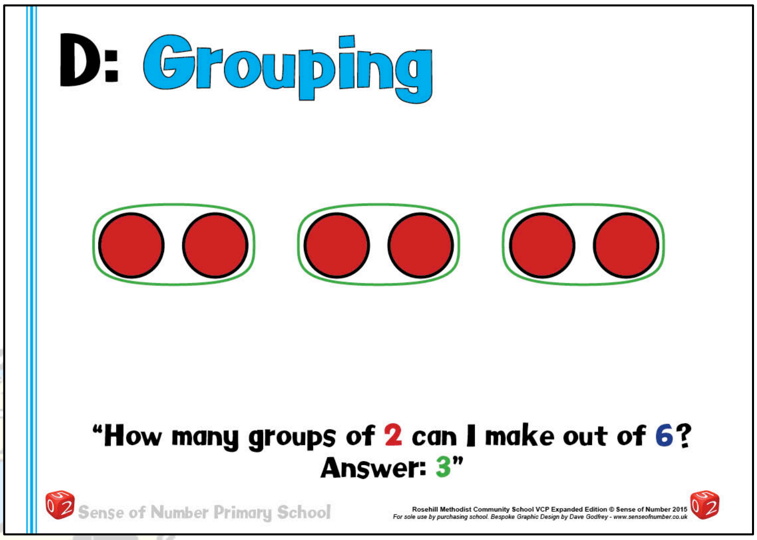
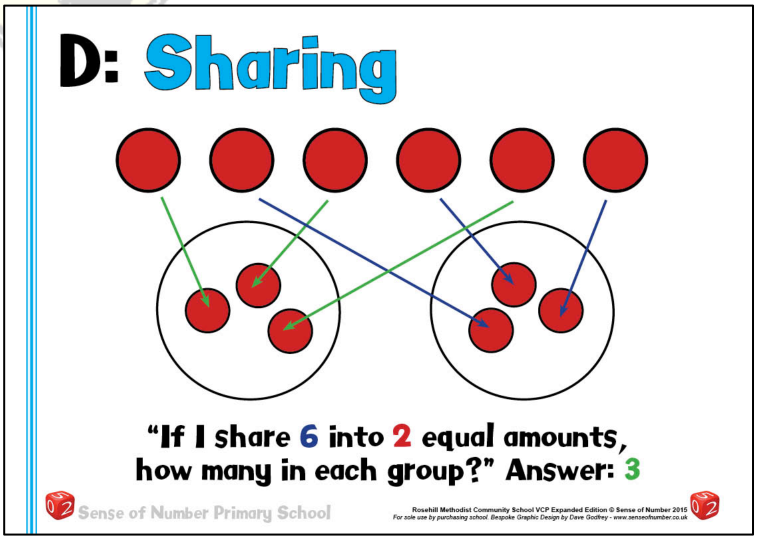
72 ÷ 4 'chunks' into 40 and 32

72 ÷ 3 'chunks' into 30 and 42 (or 30, 30 and 12)

The above points are crucial. If children do not have a secure understanding of these prior-learning objectives then they are unlikely to divide with confidence or success, especially when attempting the 'chunking' method of division.

Please note that there are two different 'policies' for chunking.

The first would be used by schools who have adopted the NNS model, the second for schools who have made the (sensible) decision to teach chunking as a mental arithmetic / number line process, and prefer to count forwards in chunks rather than backwards.

Models	Division
Grouping (The key model for division)	
Sharing (The model that links with fractions)	

Division In Key Stage 1 – Grouping or Sharing?

When children think conceptually about division, their default understanding should be Division is Grouping, as this is the most efficient way to divide.

The ‘traditional’ approach to the introduction of division in KS1 is to begin with ‘sharing’, as this is seen to be more ‘natural’ and easier to understand.

Most children then spend the majority of their time ‘sharing’ counters and other resources

(i.e. seeing 20 ÷ 5 as 20 shared between 5’) – a rather laborious process which can only be achieved by counting, and which becomes increasingly inefficient as both the divisor and the number to be divided by (the dividend) increase)

These children are given little opportunity to use the grouping approach.

(i.e. 20 ÷ 5 means how many 5’s are there in 20?’) – far simpler and can quickly be achieved by counting in 5s to 20, something which most children in Y1 can do relatively easily.

Grouping in division can also be visualised extremely effectively using number lines and Numicon. The only way to visualise sharing is through counting.

Grouping, not sharing, is the inverse of multiplication.

Sharing is division as fractions.

Once children have grouping as their first principle for division they can answer any simple calculation by counting in different steps (2s, 5s, 10s then 3s, 4s, 6s etc.). As soon as they learn their tables facts then they can answer immediately.

E.g. How much quicker can a child answer the calculations 24 ÷ 2, 35 ÷ 5 or 70 ÷ 10 using grouping? Children taught sharing would find it very difficult to even attempt these calculations.

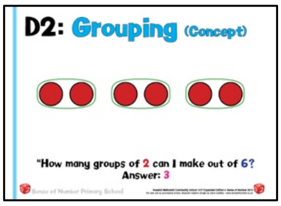
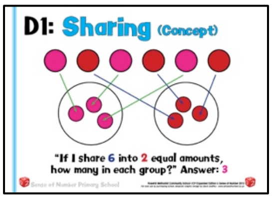
Children who have sharing as their first principle tend to get confused in KS2 when the understanding moves towards ‘how many times does one number ‘go into’ another’.

When children are taught grouping as their default method for simple division questions it means that they;

- secure understanding that the divisor is crucially important in the calculation
- can link to counting in equal steps on a number line
- have images to support understanding of what to do with remainders (Numicon)
- have a far more efficient method as the divisor increases
- have a much firmer basis on which to build KS2 division strategies

Consequently this policy is structured around the teaching of division as grouping, moving from counting up in different steps to learning tables facts and eventually progressing towards the mental chunking and ‘bus stop’ methods of written division in KS2.

Sharing <u>is</u> introduced as division in KS1, but is then taught mainly as part of the fractions curriculum, where the link between fractions and division is emphasised and maintained throughout KS2.

<table>
<tr><th rowspan="2">Stage 1</th><th colspan="2">Concepts and Number Lines (pre-chunking)</th></tr>
<tr><th>Grouping</th><th>Sharing</th></tr>
<tr><td>FS</td><td colspan="2">From EYFS onwards, children need to explore practically both grouping and sharing.
Links can then be made in both KS1 and KS2 between sharing and fractions.</td></tr>
<tr><td>Y1</td><td colspan="2">Begin by giving children opportunities to use concrete objects, pictorial representations and arrays with the support of the teacher. Use the words ‘sharing’ and ‘grouping’ to identify the concepts involved. Identify the link between multiplication and division using the array image.</td></tr>
<tr><td></td><td>
</td><td>
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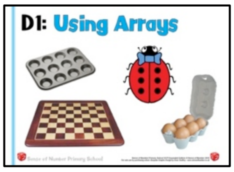
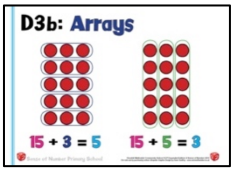
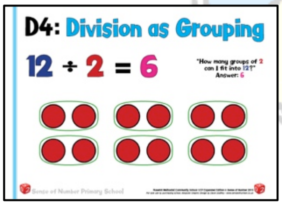
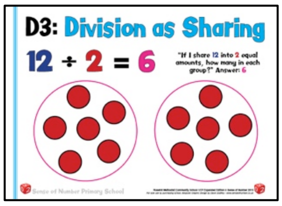
<tr><td></td><td>


</td><td></td></tr>
<tr><td>Y2</td><td>Identify Grouping as the key model for division. Relate to knowledge of multiplication facts. Use the key vocabulary: ‘20 ÷ 5 means how many 5’s can I fit into 20?’</td><td>Identify Sharing as the secondary model of division.</td></tr>
<tr><td></td><td>
</td><td>
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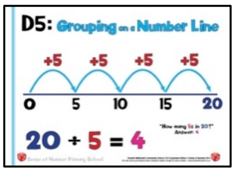
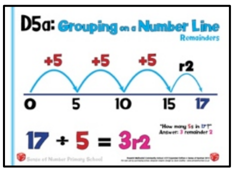
<tr><td></td><td>Counting on is the easiest route when using a number line to solve a division calculation.</td><td></td></tr>
<tr><td></td><td>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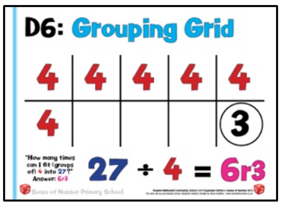
<tr><td>Y3</td><td>Continue to give children practical images for division by grouping: e.g. use PE mats and ask children to move into groups of 4. The remainder go into a hoop. Use Numicon shapes – how many 4 pieces can I fit into 27 (made of two tens and a seven piece).</td><td>Regularly stress the link between multiplication and division, and how children can use their tables facts to divide by counting forwards in steps.</td></tr>
<tr><td></td><td>
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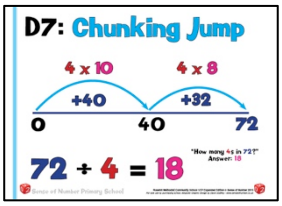
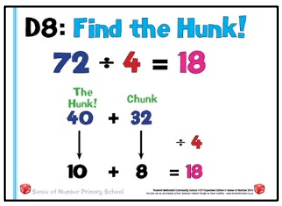
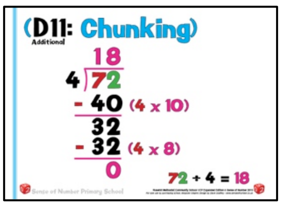
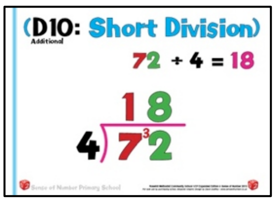
Stage 2	Chunking & Standard Methods **Chunking** **Find the Hunk & NNS Chunking**	**Standard Methods**
	As previously encountered in Y2, developing an understanding of division with the number line is an excellent way of linking division to multiplication. It can show division both as repeated subtraction, but it is simpler to show division by counting forward to find how many times one number 'goes into' another.	
Y3		
	'Find the Hunk' is a mental strategy based on mental partitioning. For the example below, the Hunk is defined as being 10 times the divisor. i.e. the divisor is 4, so the Hunk will be 4 x 10 = 40. Both chunks are then divided by the divisor and then the groups totaled. Where as 'Find the Hunk' is a mental strategy based on mental partitioning, the National Strategy chunking method is based on subtraction. Here 40 (4 x 10) is initially subtracted from the dividend. This strategy is somewhat confusing and the recommendation is to use Find the Hunk as the default strategy.	These slides introduce the Short Division (Bus Stop) method in Year 3. It is recommended that if children are introduced to this strategy in Year 3, it is only introduced at the end of Year 3 (ideally kept until Year 4) and that the key methods in Year 3 remain the use of Number Lines and the mental chucking method known as 'Find the Hunk' (see opposite) When introducing Short Division formally, use dienes (Base 10) and make sure you introduce it using the **sharing model**. The calculation starts with, 'I have 7 tens, to share between 4 people. That's 1 each with 3 remaining. These three tens are exchanged into 30, ones. The 32 ones are now shared between 4 people – that's 8, ones each.'
	 	
	Show the children examples of chunking where the quotient includes remainders.	

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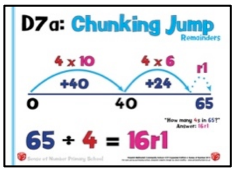

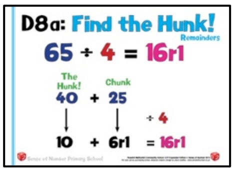

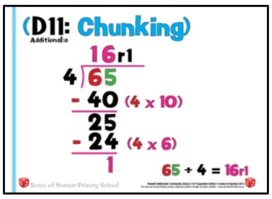

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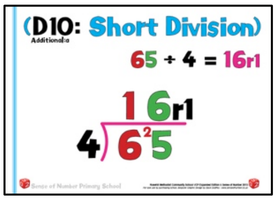

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<td>‘Mega Hunk’ is the natural development of the ‘Find the Hunk’ strategy Here Mega Hunk is defined as being multiple of 10 times the divisor. In the case below the divisor is 4, so the Hunk will be 4 x (10 x 3) = 120. Again, both chunks are then divided by the divisor and then the groups totaled.
The National Strategy chunking method is also based on the multiples of 10 times the divisor. D11b slide is an expanded version of D11. Jottings can be made to spot the multiples of 10 times the divisor (e.g. 40, 80, 120 etc.).
This strategy links to the Grouping model.</td>
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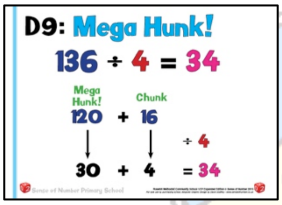

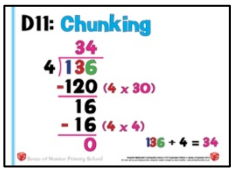

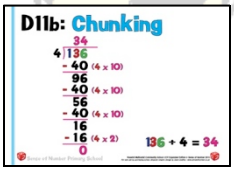

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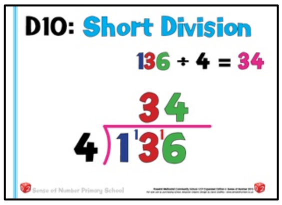

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<tr>
<td>Y5</td>
<td>Continue to use the Find the Hunk strategy whenever possible.</td>
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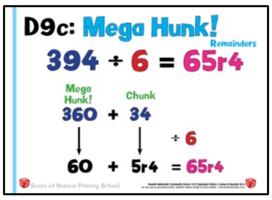

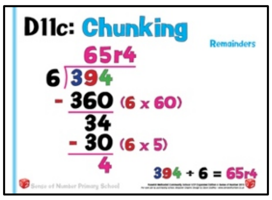

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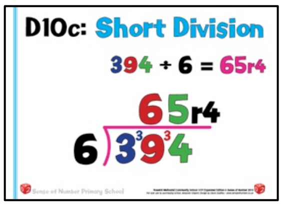

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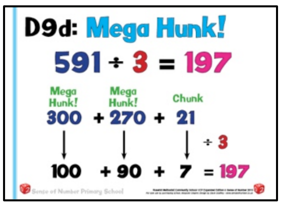

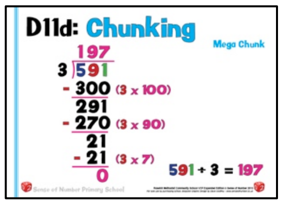

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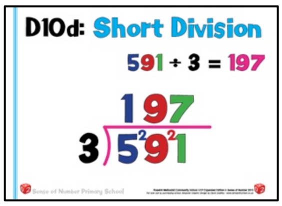

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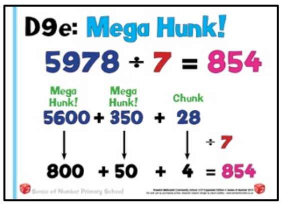
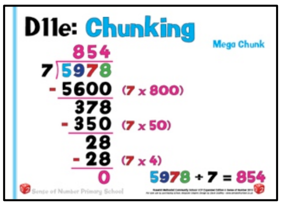
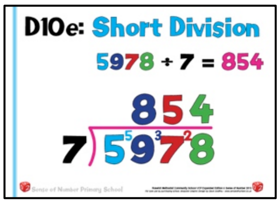
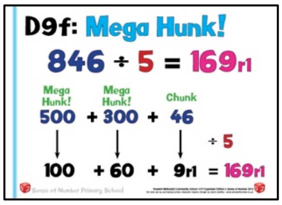
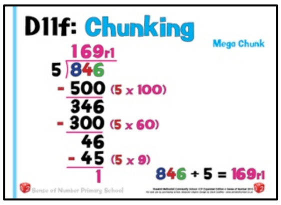
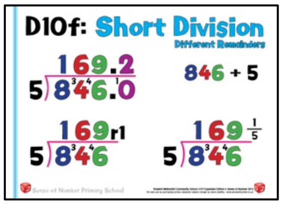
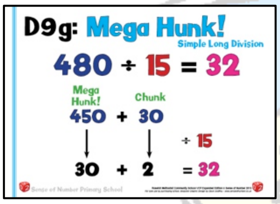
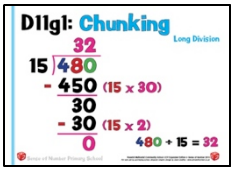
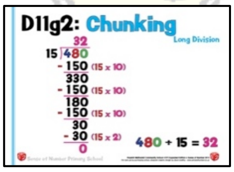
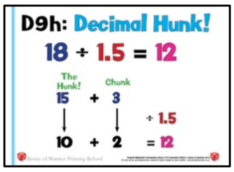
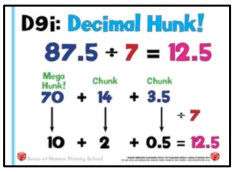
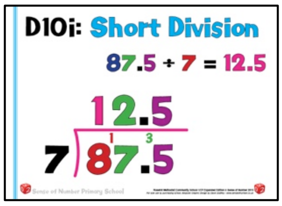
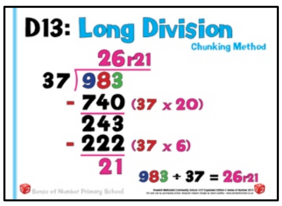
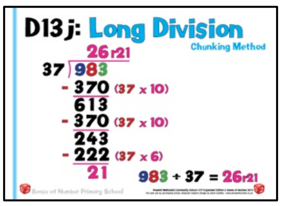
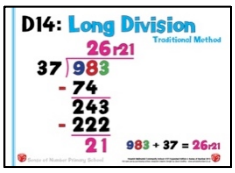
	 	
	Begin by subtracting several chunks, but then try to find the biggest chunks of the divisor that can be subtracted.	hildren should develop the ability to represent the quotient to include a straight forward remainder, but also as a decimal or fractional remainder.
	 	
Y6	When introducing long division, it is often easier to find the quotient using the Mega Hunk strategy.	
		
	 	
	 	
	There are three different ways of calculating using Long Division: The Short Division method, the Traditional Method and the NNS Chunking method. The Traditional Long Division method ignores place value, and therefore is not as helpful as the Chunking Method, which now becomes the recommended strategy.	
	 	 

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